PRINCIPLES

OF

HYDROSTATICS,

&c. &c.

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THE PRINCIPLES

OF

HYDROSTATICS;

AN ELEMENTARY TREATISE

ON

THE LAWS OF FLUIDS,

AND THEIR PRACTICAL APPLICATIONS.

BY

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PREFACE.

In the following pages I have endeavoured to exhibit the Laws of Fluids, and the principles of their practical applications in a form adapted to all students of Mathematical and Natural Philosophy.

Many very valuable treatises have been published on this subject; but from their being either almost exclusively practical or exclusively mathematical, and from the physical parts of the science being detached from the theoretical, the student cannot derive much real advantage from them, unless previously acquainted with the principles of the science, and with the Differential and Integral Calculus; for unless so previously prepared, he must content himself with adopting results without being thoroughly acquainted with the processes by which they are obtained. It is abvious that this method of arriving at an end, without understanding the means, may employ, but cannot properly instruct the mind.

I have endeavoured in the following treatise to develop the principles of the science with the use of none but the most elementary mathematics; so that the student, who now either partially or wholly neglects this beautiful branch of Natural Philosophy, from the uninviting character which Analysis presents to those who are not familiar with it, may proceed to its study, with the knowledge of a few propositions of Geometry, Algebra and Mechanics, and for understanding by far the greater portion of the subject even that amount of previous knowledge is not necessary.

The plan here adopted is similar to that which has been pursued by Dr Whewell in the kindred science of Mechanics;

and being already peculiarly indebted to him, not only on account of the advantage which I derived from his college lectures upon Hydrostatics, but for his advice at the commencement of this work, I feel pleasure in laying myself under additional obligation, by availing myself of his language in illustration ot the plan here attempted to be pursued.

"These principles may be familiarized to many who may never become dexterous analysts, and there are advantages in not introducing the general analytical methods till some of the more simple cases have been separately considered; for these methods suppose a command of analysis which can only be acquired in time, and which therefore should not be taken for granted at an early period. And it is desirable that this science should be studied by persons who do not ever acquire a knowledge of more than the elementary parts of mathematics.

"In a system of study like that of our University, it would be a mistake to present the introductory subject in a manner which supposes that the learner is necessarily to advance far in mathematical pursuits. We ought to lay our foundation so as to admit of such a superstructure, but not so as to be useless without it*."

It has been truly observed that the whole science of Hydrostatics in its most modern form is only the development of the idea of fluidity in connexion with pressure†; but how difficult it is for the mind to apprehend that idea in such a manner as to deduce from it all the remarkable conclusions constituting the science of Hydrostatics, the history of the science from the time of Stevin and Galileo and daily experience fully shew; and hence it is that the apprehension of that idea, and the connected chain of propositions whereby it is fully developed, may afford scope for the exercise of much thought, and may

* Preface, *Elementary Mechanics*.

† See *History of the Inductive Sciences*, Vol. I. page 102. Second edition.

become a most valuable agent for the purposes of general education, and for training and disciplining the mind and familiarizing it with the process of inductive reasoning.

The general propositions, as well as the fundamental principles or axioms of the science are few; but the division of the subject necessary for presenting it in a practical form gives rise to several specific propositions which are strictly applicable only to some particular cases. But though the general propositions of the science are few in number, their practical applications and illustrations are very numerous, as a glance at the annexed table of contents will shew.

It is not the object of the following pages, nor are the machines and illustrations introduced, intended to teach and represent practical engineering: the student who wishes to inform himself as to the precise details of construction of the steam-engine, for instance, and of other machines, as practised by the mechanical engineer of the present day, must have recourse to treatises especially devoted to practical science, and in many cases to the workshop: my object having been to shew the principles and modes of action which are common to all machines of the particular class referred to, rather than to shew the precise manner in which an engineer would construct them; hence, with some few exceptions, the precise details of the construction form no part of the descriptions contained in the following pages. He who is acquainted with the general principles and mode of action of the particular machine, will have little difficulty in tracing and referring to its true cause any peculiarity which may present itself in the construction of the mechanical details.

The student who shall have made himself acquainted with the general principles of the science, and the Differential and Integral Calculus, will find in my *Theory of the Equilibrium and Motion of Fluids*, or in other works, the development of the laws of fluids by the power of mathematical analysis.

In this edition I have added a chapter on Waves and Tides, and the Equilibrium and Motion of Water in Tidal Rivers and Estuaries, subjects which have hitherto not received that attention which their importance deserves. No portion of the subject presents more instructive illustrations, and on few subjects do so many erroneous impressions exist. I have also introduced throughout the work such additional illustrations as are suited to fix attention on the principles involved in the practical applications of the Laws of the Equilibrium and Motion of Fluids. I take this opportunity of thanking those who have favoured me with suggestions upon the work, and I shall be greatly obliged by any hints whereby its utility for the purposes of University or general education may be increased.

T. W.

LONDON,
May, 1856.

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THE

PRINCIPLES OF HYDROSTATICS.

CHAPTER I.

ON THE PROPERTIES OF FLUIDS.

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1. THE substances presented to us by nature may be divided into solid and fluid matter. Now all matter is subject to certain laws, according to which, the phenomena of motion and of rest, of pressure and of the other effects arising from the mutual action of bodies and of their particles, take place; the investigation of the laws which express these phenomena is the chief object of Physical Science.

The laws, to which solid matter is subject, are treated of in Mechanics; and it is the object of that part of Natural Philosophy, which is commonly termed Hydrostatics, to investigate these laws for fluid matter, that is, to determine the conditions according to which liquids and gases are at rest or in motion, and to shew the practical application of these conditions as exhibited in the pressure of fluids, in the support of floating bodies, in various instruments and machines whose use and action depend on the general laws of fluid equilibrium and motion, and in a vast variety of natural phenomena.

The particles which compose solid bodies or combine into dense and hard masses, are held together by a force, termed the force of cohesion, which either does not exist at all or exists only to an extremely small and almost inappreciable amount in fluids. Now all those substances

in which this force exists in so slight a degree, if it exists at all, that the particles can be moved amongst each other with a resistance scarcely appreciable, are called fluids, and the laws of their equilibrium and motion are treated of in the following pages.

The laws by which matter is governed in the constitution of bodies, present a series of the most interesting and intricate questions. Among these the infinite divisibility of matter has been much agitated. To determine this by direct experiment is, owing to the imperfection of our senses, hopeless. The argument in favour of this hypothesis is derived from pure Geometry. For a line or surface admits of division without limit, and to whatever degree matter is divided, since each small particle possesses surface and extension, it may be considered susceptible of still further division*. Many objections might be urged against this conclusion, but the researches of Dalton, and the discoveries of modern Chemistry, furnish strong arguments against the infinite divisibility of matter, and in favour of the Atomic Theory. For all the changes which we can produce consist in separating particles that are in a state of cohesion or combination, and in joining those that were previously at a distance: no creation or destruction of matter is within our power, or the reach of chemical agency. And since the chemical properties of the particles would be necessarily altered by a change in their form and magnitude, but no such alteration is ever observed in their properties, we may conclude that there are certain dimensions beyond which matter cannot be reduced.

The Atomic Theory of Dalton leads us, therefore, to assume that bodies consist of ultimate particles or atoms which are indivisible by any means mechanical or chemical within our reach. These atoms may indeed be conceived

* See Webster's *Elements of Physics*, p. 11.

to be divisible, but are incapable of actual division by our agencies. We have only to assume with Dalton that all bodies are composed of ultimate atoms, the weight of which is different in different kinds of matter, and we explain at once the laws of chemical union; and this mode of reasoning is in the present case almost decisive, because the phenomena do not appear explicable on any other supposition*.

2. *Fluidity.* The facility with which the component particles of a fluid are moved amongst each other constitutes the characteristic distinction between solids and fluids. For the particles composing a solid mass cannot be readily separated and moved amongst each other, and in general their separation can only be effected by the application of an appreciable amount of force; whereas it is difficult to conceive any force, however small, which is not sufficient to separate and move amongst each other the particles of a fluid mass. It is however evident that the particles of different fluids are moved amongst each other with different degress of facility; whence the term *fluidity* is applied to fluids, by which is meant the degree of facility with which the particles may be separated or moved amongst each other. Hence fluids have been divided into perfect and imperfect; under the former of which are classed those whose component particles have no sensible cohesion, and under the latter, those whose particles have some sensible cohesion. This distinction is hypothetical and arbitrary, since no fluid possesses perfect fluidity; and the terms perfect and imperfect indicate only certain states in which the fluid exists: thus the fluidity is very perfect in boiling water and very imperfect in oil, and the viscidity or tenacity of any particular fluid appears to increase as the temperature decreases.

* See Turner's *Chemistry*; Dalton's *New System of Chemical Philosophy*; Daubeny's *Atomic Theory*.

3. *Fundamental Properties of Fluids.* Our own observation and experience make known to us the three following fundamental properties of fluids. 1. That a fluid can be readily divided in any direction. 2. That its parts press against each other and against any solid surface with which they are in contact. 3. That its parts press, that is to say, transmit pressure equally in all directions.

It will be seen hereafter how these which are the fundamental properties and characteristics of all fluids may be derived from the definition of a fluid.

4. *Definition of a Fluid.* A fluid is a collection of particles indefinitely small and capable of acting on and moving amongst each other in every direction without friction.

The above definition of a fluid suggested by the facts which observation and experience teach us will serve as the basis of our mathematical reasoning on fluids. It will follow, as a corollary from the above definition, that fluids may be readily divided in any direction, and it will be seen hereafter in what manner the fundamental properties of the transmission of pressure (Art. 3) may be derived from the same definition*.

5. *Liquids and Gases.* Fluids are divided into liquids and gases, and the characteristic difference betwixt these two substances is, that a slight attraction exists between the particles of a liquid, for they collect under certain circumstances into spherical drops, but between the particles of a gas, a repulsive force exists, in consequence of which a gas dilates or increases in volume, unless confined to a certain bulk by pressure. Thus a liquid may be defined as a fluid having no tendency of itself to increase

* Professor Challis gives the following definition of a fluid: "A perfect fluid is one which admits of being divided by an indefinitely thin solid substance without any assignable force."—*Syllabus of Lectures*, Art. 1.

in bulk; and a gas, a fluid having such tendency. Also we may consider the forces of mutual attraction and repulsion betwixt the particles as counterbalancing each other in a liquid, whereas in a gas the force of repulsion predominates.

It is convenient also to consider gases as divisible into permanent gases and vapours. Some substances present themselves under three forms or states, namely, solid, liquid, and gaseous; others under two, and others under one. Thus water may exist in three different states, as ice, as a liquid, and as a vapour; but alcohol has hitherto been known to exist only as a liquid and a vapour; and common air only as a gas; hence air is said to be a permanent gas.

Those gaseous substances which do not readily assume the liquid state are called permanent gases, and those which readily assume that state are termed vapours*.

Fluids are also distinguished by the terms elastic and inelastic, the former of which includes the gases and the latter the liquids. It will however be seen hereafter that all fluids are elastic, or undergo some change in density on a change in the pressure to which they are subject.

6. *Dilatability.* By the dilatability of a substance is meant that quality in virtue of which it has a tendency to increase in bulk without any increase in the quantity of matter. It is this quality which gives rise to the division of fluids into liquids and gases (Art. 5), the former of which do not readily change in bulk, but the latter increase in bulk, or dilate, as the pressure to which they are subjected decreases.

7. *Effect of Heat.* The influence of this agent in the different physical states of bodies, as solid, liquid, or gaseous,

* It has been ascertained by Dr Faraday and others that many of the gases which are ordinarily called permanent gases may by cold and pressure be reduced to liquids. See *Phil. Trans*, 1823, pp. 160, 189; and 1845, p. 155.

is very remarkable. All bodies expand by heat and contract by cold, but the law of the expansion and contraction is very different for solids and fluids. By the increase of beat solids may be converted into liquids and liquids into vapours or gases, and on its decrease the liquids are reconverted into solids and the vapours or gases into liquids. Thus it appears that cohesive force is weakened and destroyed by the agency of heat, and repulsive force is called into action.

To this general law however as to the effect of heat and cold in causing bodies to expand and contract there are some remarkable exceptions; thus water as it passes into ice expands; some of these peculiarities will be referred to hereafter*.

8. Prop. *Every particle of matter, fluid as well as solid, has weight.*

It is evident that any considerable portion of any fluid matter, such as water, is heavy; and the weight of exceedingly small portions may be accurately determined: whence it may be inferred that every particle (every elementary particle on the hypothesis of constituent atoms, Art. 1) has a certain definite weight, though not appreciable by our senses. A similar inference may be made respecting air and all the gases, for the actual weight of very small portions even of the lightest of them may by nice experiments be determined, whence as before the weight of every portion or collection of particles is inferred, and thence the weight or gravitation of every particle of the fluid matter.

9. Prop. *The particles of all fluids gravitate, the upper parts pressing on the lower.*

The consequences of the weight or gravitation of the particles of a fluid, and the attempted explanation of some well-known phenomena, gave rise, in the infancy of the

* See Chapter on Temperature and Heat.

science of Hydrostatics, to objections both interesting and instructive.

That the weight or gravitation of a fluid mass is the consequence of the gravitation of each individual portion of that mass somehow or other collectively effective, could never be disputed. But that each particle does singly and separately gravitate, and act on the one immediately in contact with it, so as by this united action collectively to compose the weight of the whole, was for a long period the subject of much discussion*. The fact that a bucket full of water weighs much less when immersed in water than when not immersed, was appealed to, and the inference was that the water in the bucket does not gravitate because it is surrounded by its own element, so long as the bucket is immersed in the water.

Again, appeal was made to the case of divers, who are said to experience no additional pressure though they descend to great depths, but move about without constraint and as freely as when near the surface; whereas if fluids do gravitate in *proprio loco* the divers must sustain considerable pressures. Now the fact is, that divers do sustain additional pressure; but owing to the equality of pressure on all sides no distortion or dislocation can take place; and this fact was fully established by Boyle, in experiments on tadpoles, which moved about with perfect facility, though their bodies were considerably reduced in size by the fluid pressure. But the depths to which divers descend is very small when compared with the depths to which any heavy substance may be sunk; and it is a well-known fact, that if an empty bottle be corked and sunk to the depth of fifty fathoms, either the cork will be forced in or the bottle broken. The diminished weight of

* The action of Gravity was supposed to be the attempt of a body when out of its place to get into it. Hence when a body was not out of its place there was no reason for gravity to exert itself, and therefore bodies did not gravitate *in proprio loco*, or when they were in their proper place, as a portion of fluid surrounded by its own element.

the bucket will be shewn also to be the necessary consequence of the gravitation of fluids in *proprio loco*. Many other objections were raised and facts appealed to, all of which however may be readily explained; and the curious reader will find full details respecting them in the philosophical works of Boyle*.

10. *Action of Gravity.* From the last two propositions it is evident that the particles of fluids are subject to the same law of gravity as the particles of solid bodies; and if experiments were made on the descent of small masses of fluids in the same way as experiments are made on solids by Atwood's machine, or in any other manner, it would appear that the force of gravity accelerates the motion of a falling fluid mass, adding equal velocities in equal times, precisely according to the law by which it accelerates a falling solid body. In confirmation of the assertion, that the particles of a fluid mass are subject to the same law of gravity as those of a solid, we may instance the construction of pendulums with mercurial bobs, that is, the bobs instead of being a solid mass of metal, are hollow vessels filled with mercury; and any other fluid if heavy enough would do equally well, so far as the mere oscillation of the pendulum is concerned.

11. *Measure of fluid pressure.* Fluid pressure like all mechanical force is measured by weight; and gravity (being a force which under similar circumstances acts on all bodies equally, so as to produce the same effects on all, that is, generating the same velocity in all in the same time,) is most conveniently measured by the velocity generated in a given portion of time, as 1″. Now velocity is measured by space passed over, and the velocity generated in a given time is measured by the space which would be passed over in an

* See Vol. II, p. 786. Ed. Lond. 1772.

equal portion of time by a body moving with the velocity generated during that time. Hence the velocity generated in 1″ is measured by the space which a body, moving with that velocity, would pass over in 1″.

Let this space be represented by the letter g, and g is from numerous experiments found very nearly equal to $32\frac{1}{5}$ feet*. This then is the measure which is adopted, and the force of gravity effective in producing motion or pressure, or, as it is called, the accelerating force of gravity, is measured by this quantity.

12. *Density.* A substance may contain a greater or a less quantity of matter in a given volume or bulk, according to the circumstances under which it is placed, that is, it may be more dense or have a greater density under some circumstances than under others. If then some uniform standard or unit of volume be assumed, we may define the density of a body to be the quantity of matter contained in that unit of volume. The Greek letter ρ will be employed in the following pages to represent the quantity of matter in a unit of volume, or the density of the body.

13. *Measure of the weight of a body.* The weight of a body, or the force which must be exerted in consequence of the tendency of a body to fall to the earth, to prevent its so falling, is caused by the accelerating force of gravity acting on all its particles, and will therefore be proportionate to the quantity of matter which the body contains. Hence, if g be the force of gravity (Art. 11) on one particle, and ρ be the quantity of matter in the unit of bulk (Art. 12), the whole weight of that unit will be $g\rho$.

Now every body is measured by the number of units of bulk which it contains; the weight therefore of one unit being known, the weight of the whole body is known also.

* Whewell's *Elementary Mechanics.* Sixth edition, Art. 81.

Let V be the volume of a body, that is, the number of units of bulk which it contains, then $Vg\rho$ is the weight of the body.

14. *Compressibility.* By the compressibility of bodies is meant that quality in virtue of which a body admits of its bulk being reduced by pressure, so that an equal quantity of matter may be contained in less space.

Hence when a body is compressed its density is increased. (Art. 12.)

15. *Elasticity.* By the elasticity of bodies is meant that quality in virtue of which a body resists compression or extension, and resumes its original state when the force to which it was subjected has ceased to act.

In treating of the elasticity of bodies as a practical question, care must be taken to distinguish between the elasticity which is attended with a change of volume, and that which is attended with a change of form; both these changes generally co-exist. When a quantity of air is compressed by a piston in a cylinder, or by a tumbler inverted over water and pressed down, the elasticity is accompanied by and proportional to the change of volume, as will hereafter appear (Art. 89), but when a steel spring is bent, the elasticity is accompanied by a change of form; in the former case the change of volume is the principal feature to be remarked, and in the latter the change of form. The well-known substance caoutchouc, or Indian rubber, will afford a good exemplification of the elasticity connected with both change of form and change of volume. If a ball of this substance be compressed, or drawn out and elongated, its form readily changes, and it resumes its original shape and size when the force of compression or of tension to which it was subjected is removed, whereas if the substance be enclosed by rigid sides the elasticity is materially affected. This is the case with a drop of water or mercury; if lightly pressed it changes

its form, and returns to it immediately on the pressure being removed, but water or mercury, if enclosed in a tube and pressed by a piston, is under ordinary pressures inelastic and incompressible.

16. *Perfect Compressibility and Elasticity.* The qualities of compressibility and elasticity are found in different bodies in very different degrees. A substance is said to possess perfect compressibility, when the diminution of its volume is proportional to the compressing force, and perfect elasticity, when the force of restitution is equal to the force of compression.

Hence, in a perfectly elastic body, the elastic force is proportional to the diminution of volume. Such appears to be the case with common air.

17. PROP. *All Fluids are compressible and elastic.*

The compressibility and elasticity of the gases could never have been disputed, and they consequently obtained the name of the elastic fluids, but the liquids exhibit so little appearance of yielding to compression that they were formerly considered as incompressible and inelastic.

Florentine experiment. The incompressibility of water was considered as established by the experiment of the academicians of Florence near the end of the 17th century. In this experiment a sphere of gold filled with water and accurately closed up was subjected to pressure, when some of the water appeared like dew on the outside. Now the solid content of a sphere is greater than any figure of equal surface; hence no change could take place in the form of the sphere on its yielding to the pressure without its capacity being diminished and the water being compressed. But the sphere having changed its form, and the water having consequently been forced through the pores, it was concluded that water is absolutely incompressible. This experiment

can only shew that water may be forced through the pores of a mass of gold more easily than it can be compressed. And had the water not escaped in this way, but appeared to yield to the pressure, and to occupy a smaller space, its compressibility could not from this circumstance have been inferred, unless it was certain that the sphere had not yielded in any part so as by some extension in the materials of which it was composed to make up for the diminution in solid content consequent on the change in its form.

Canton's experiment. The opinion of the absolute incompressibility of water prevailed very generally until Canton proved its incorrectness, by measuring with great accuracy the compressibility of water and other liquids. The experiment of Canton was at once simple and decisive. A glass tube of small diameter, and carefully graduated, having a large bulb at the bottom, was filled to a certain height with water; on placing this under the receiver of an air-pump and exhausting the air, the water rose in the graduated tube, and sunk again to its original height on the air being readmitted. Also, on placing it in the receiver of a condenser, and subjecting it to an increased pressure the water sunk in the tube. Thus, the degree of compression of the water due to the weight of the atmospheric pressure on the surface of the glass and of the water was ascertained, and Canton assigned the compression of water for the pressure of an atmosphere at $\cdot000044$, or $\frac{1}{22727}$ of its volume*. The preceding experiment establishes also the elasticity of water, since the water returns to its original bulk on the increased pressure being removed.

Perkins's experiment. The compressibility of water was proved in a different manner by Mr Perkins, namely, by

* See *Philosophical Transactions*, 1762 and 1764.

subjecting water to mechanical pressure. A flask or long tube is furnished with a cylindrical stopper, and an index for measuring the amount by which the stopper may have been pressed in. This index may be a spring-ring placed on the stopper or in the neck of the flask or tube, and adapted so as to stay at whatever point it is placed. The flask or tube being filled with water up to a certain point in the neck, the stopper inserted, and the index adjusted, so that if the stopper be pressed in the index will move also, the flask is placed in a cannon or other strong vessel full of water, and more water is forced into this vessel by a pump. The water in the flask will thus be compressed, and the stopper forced in, and the index will tell the amount of the compression. The stopper returns to its original position so soon as the additional pressure is removed; thus the compressibility and elasticity of water or any other liquid may be proved, and the degree of compression accurately measured*.

The experiments of Canton have been repeated with great care by Œrstead, and his results verified in a remarkable manner. Œrstead confirms the result that water is not so compressible in summer as in winter, or at high as at low temperatures; he finds that the differences of volume in the compressed water are proportionate to the compressing power, and that the decrease of volume produced by pressure preserves the same proportion to the pressure as far as 65 atmospheres: he assigns the compressibility of water under a pressure equal to that of the atmosphere at .0000461 of its volume †.

Messrs. Colladon and Sturm have investigated the compressibility of different liquids with great precision, and they

* See *Philosophical Transactions*, 1821.

† See *Report of Third Meeting of British Association*, p. 353.

assign the following as the amount of compression due to the pressure of one atmosphere*.

Water...............	0.00004965	$\frac{1}{20148}$
Alcohol.	0.0000916	$\frac{1}{10911}$
Mercury	0.00000338	$\frac{1}{295857}$

19. While however the elasticity and compressibility of fluids, apparently inelastic and incompressible, is thus established, it must be remembered that the pressure to which they are subjected is great, and such as can rarely occur; extreme cases therefore of this kind may be disregarded in a practical consideration of the laws of fluids. Hence, for all practical purposes, fluids may be divided into elastic and inelastic. Elastic fluids being those whose dimensions are increased or diminished exactly according to the diminution or increase of pressure to which they are subjected (Art. 16); and inelastic fluids those whose dimensions undergo no sensible change for any small increase or diminution in the pressure to which they are subjected.

Common air will in general be taken as the representative of elastic and water of inelastic fluids†.

* The results of these philosophers differ in amount from those of Œrstead; the discrepancies have been referred to the compression of the tubes or vessels containing the liquid. See M. Poisson in *Mem. Ac. Sci.* 1827, 1828.

† The student is referred to the *Elements of Physics* by the Author of this work, for a more detailed account of the properties of bodies.

CHAPTER II.

ON THE EQUILIBRIUM OF FLUIDS SUBJECT TO PRESSURES.

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20. In the preceding Chapter the general properties of fluids have been treated of; we shall now proceed to consider their mechanical properties, that is, the conditions of their equilibrium when subject to mechanical action. And it is of great importance to form an accurate conception of the mechanical properties of a fluid considered as a collection of particles which may be moved amongst each other by any assignable force. We shall treat in the present Chapter of the laws of equilibrium of a fluid subject to pressure.

21. *Transmission of Pressure.* The characteristic distinction betwixt fluids and solids is, that the former transmit pressure in every direction, the latter in one direction only, namely, that in which the force is impressed. The transmission of pressure in every direction is involved in our conception of fluidity; we are perfectly convinced, on grasping any flexible vessel containing a fluid in our hands, that every portion of the containing surface experiences some pressure; that the fluid presses out in every part in consequence of the force impressed upon it by our hands.

Hence, in treating of the equilibrium of fluids, the transmission of pressure is the contradistinguishing property between them and solids. This characteristic property, involved as it is in our conception of a fluid mass subjected to pressure and remaining in equilibrium, may be considered

as the necessary consequence of the application of pressure to such a collection of particles as constitute a fluid. For all the particles being equally free to move in all directions, if any number of these particles, that is, any portion of the fluid be subjected to pressure, the particles so acted on will be immediately put in motion, unless the action be counterbalanced by the action of the contiguous particles: this mutual action will extend throughout the whole fluid mass, that is, there will be a pressure transmitted in every direction.

And further, this transmission of pressure throughout the entire mass of the fluid takes place *instantaneously*, that is, no appreciable interval of time elapses between the impression of the force and its action at every part of the entire mass. This remarkable property of fluids, namely, the instantaneous transmission of pressure equally in all directions, is the subject of many useful practical applications.

22. *Action and Reaction.* The laws of mechanical equilibrium are the same for all substances, and do not depend on the degree of coherence of the particles of the substances which are the subjects of those laws. Thus if any fluid mass subject to pressure be in equilibrium, the conditions will be precisely the same as in the action of forces on solid bodies. For the equilibrium being established, the system of particles, whether composing a compressible or an incompressible fluid, may be supposed to become rigid, without in any manner affecting the equilibrium. Now when bodies act on each other mechanically, the equality of pressure is a universal statical truth, and in order to distinguish the two actions between which the equality subsists, it is expressed by saying that *action and reaction are equal.*

23. Prop. *A fluid being subject to pressure, the transmitted action is equal to the original action.*

The action or pressure which is transmitted (Art. 21) will be transferred to every point of the containing surface. At any point in the containing surface the transmitted action will be balanced by the reaction of the surface. We have then two forces of precisely the same kind impressed on the fluid, namely, the original action and the reaction of the surface. Now this reaction, whatever be its magnitude, will, since it is a pressure impressed on a fluid, give rise to a transmitted action in all directions; and at the point at which the original action is impressed, the original action becomes the reaction; for to suppose it either greater or less, involves an absurdity. The original action then and the reaction at any point being thus convertible and equal, and the reaction being, by the general law of the equality of action and reaction, equal to the transmitted action, the transmitted action is also equal to the original action.

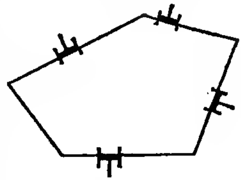
Hence a pressure exerted on a fluid is transmitted equally in all directions; thus fluids *press* in all directions, they also *press equally* in all directions.

The preceding is also true for fluids whose particles have sensible tenacity or viscidity, the only difference being, that the pressure is not transmitted in all directions with the same velocity with which it is transmitted in the direction of the impressed action. This deviation is however only instantaneous, and when the equilibrium is established the law of the equality of pressure obtains*.

24. Prop. *A fluid mass subjected to pressure, is equally pressed on all equal portions of the containing surface whatever be their directions.*

* This fundamental law of the transmission of pressure equally in all directions may be strictly derived from the definition of a fluid. See *Syllabus of Lectures*, by Prof. Challis, p. 31.

Let any vessel having any number of equal portions of its surfaces removed and replaced by pistons, as represented in the figure, be filled with fluid. Let any one of these pistons be pressed in with a given force P, then all the other pistons will start out, and the fluid will not be in equilibrium, unless a force equal to P be applied to all of them.



For the pressure exerted on the fluid is transmitted in every direction, and the transmitted action is equal to the original action (Art. 23), and the transmitted action being exerted on the bases of pistons which are all equal, must exert equal pressures on all; and the forces which must be applied to each of these pistons to counterbalance the transmitted action, must, since it is to be applied in the same manner and to pistons equal to the first piston, be equal to the force which produces the original action. Hence a force equal to P must be applied to all the other pistons, that the equilibrium may be maintained. Now these pistons may be in any parts of the surface of the containing vessel, and of any magnitude; and the vessel may be of any conceivable shape; whence it is evident that all parts of a fluid in equilibrium are by the action of a force equally pressed on all equal portions of the containing surfaces.

25. *Unit of Pressure.* The action exerted on any portion of the surface of the containing vessel is proportional to the area of that portion. For in the preceding proposition the action exerted by the transmitted pressure, being counterbalanced by the pressure produced by a force equal to P applied to each of the pistons, is equal to that pressure. Let the bases of each of these pistons be equal to the unit of area; then the pressure exerted on a portion of the fluid

equal to the unit of area being P, that exerted on a portion containing A units will be PA; for the given surface may be replaced by a number of pistons equal to the number of units of area which it contains, and the pressure on each of these being P, the pressure on the whole area is PA, which is proportional to A. In estimating the pressure transmitted to the surface of vessels by the action of forces whose magnitude is known, it is convenient to have some standard to which every case may be referred. The pressure exerted on a unit of area is, for obvious reasons, a convenient one, hence the pressure at any point is measured by the pressure which is or would be exerted on a unit of area situated at that point. And the pressure so exerted will be denoted by the general symbol (p). From this reference to a unit of area the quantity p is termed the *unit of pressure*. The unit of area may be a square inch, or any other convenient measure.

26. *Illustrations.* The preceding reasonings apply equally to all fluids whether compressible or incompressible. For the equilibrium being once established, the nature of the connexion which exists among the particles is quite immaterial, since the whole system may be supposed to become rigid (Art. 30).

Several illustrations of the preceding propositions present themselves in machines which are in constant use. If a fluid were not equally pressed on all equal portions of the containing vessel, the use of safety valves could not be depended on. For a pressure which would burst a vessel might exist in one part without being indicated in every other part. Hence unless the safety valve were placed at this particular part, it would afford no security against the bursting of the vessel, whereas it is known as a fact, ascertained by daily experience, that the safety valve may be placed anywhere.

Thus in the Bramah Press* (Chap. XI.), the safety valve *L* may be anywhere between the forcing valve *F* and the working cylinder, that is, at any part of the containing vessels which is in free communication with the fluid subject to the pressure which may become greater than the strength of the materials can bear, since from this property of fluids it is certain, that wherever the safety valve is placed, if it be not too heavily loaded, the extreme pressure will be relieved by the escape of some of the fluid through the valve before the pressure is sufficient to burst the vessel.

Again, the safety valve of steam-engine boilers may be at any part of the surface with which the steam is in contact, provided that part be always in free communication with the steam in the boiler; or, as is usually the case, it may be at the end of a pipe just above the boiler, since it is certain that the elastic force of the steam will exert the same action on the valve that it exerts on every portion of the boiler equal to the area of the valve. Suppose that a steam boiler is constructed to bear a pressure of 20 lbs. on every circular inch of its surface (that is, on any portion of its surface equal to a circle of an inch in diameter) without any danger of bursting. Let a safety valve whose area is one inch in diameter be placed at any part of the boiler and loaded to 20 lbs., that is, so loaded that it will require an upward pressure of 20 lbs. to raise it. Then if the elastic force of the steam be such that it will exert a pressnre of 21 lbs., on every circular inch, the valve will open, and the steam will escape till its elastic force is diminished by 1 lb. on every circular inch, when the valve will immediately close.

Bellows for blowing furnaces are made of very various constructions, and it is perfectly immaterial, so far as the magnitude of the blast is concerned, in what part of the

* The student may proceed at once to the Chapter on the Hydromechanical or Bramah Press.

bellows the aperture for the tube, which conducts the air to the furnace, is made.

The preceding furnish good illustrations of the truth of the principle of the *equal transmission of pressure*, or of the *equality of pressure*, in those fluids with which we are best acquainted; and the same would be found to be true in the practical application of other fluids *.

27. Prop. *Forces having any assignable ratio to each other may, by the transmission of pressure in a fluid, be in equilibrium.*

If a vessel full of fluid have any number of equal portions of its surfaces removed and replaced by pistons, and a pressure be exerted on the fluid by a force applied to one of these pistons, the equilibrium cannot be preserved unless an equal force be applied to all the other pistons (Art. 24). Let a force P be so applied, that any one piston may be considered as keeping all the rest in equilibrium.

Let the area of the bases of each of the pistons be equal to A, and let the force P applied to the first piston be supposed to preserve the equilibrium on a number of pistons equal to n. Then since these n pistons may be situated anywhere, let them be contiguous to each other, so as to make up a portion of the surface of the vessel equal to nA.

Then the equilibrium subsists on this fluid machine by the transmission of the pressure between a force P applied to an area A and a force nP applied to an area nA. Let

$$nP = Q, \text{ and } nA = B.$$

* This property of the instantaneous transmission of pressure admits of various other useful applications, as for instance to the transmission of signals and telegraphic purposes. An ingenious application was made some time ago by Mr Francis Whishaw, to an Hydraulic Telegraph, in which by the rising and falling of a float in two similar cylindrical vessels, containing water and connected by a small pipe, a very extensive set of signals might be communicated between distant places.

Let p be the unit of pressure. Then $P = pA$, $Q = pB$;

$$\therefore \frac{P}{A} = p = \frac{Q}{B}, \text{ and } Q = \frac{B}{A} P.$$

The ratio therefore of the forces Q and P, which are in equilibrium on this fluid machine, depends entirely on the ratio $\frac{B}{A}$; and by properly assuming this ratio the forces Q and P may have any assignable ratio whatever. The application of this principle in Bramah's Press has led to a machine which is practically as well as theoretically one of the most perfect and efficient with which we are acquainted. In this press a weight of several thousand tons may be balanced by the weight of a single man.

28. It must be carefully borne in mind, that in the preceding articles of this Chapter no reference has been made to the weight of the fluid mass, or of its component particles; but the mass of fluid has been treated as a collection of particles devoid of weight, and not subject to the universal action of gravity. The preceding propositions, consequently, will apply equally to the lightest and heaviest fluids, to atmospheric air and the permanent gases, as well as to water and mercury, to whatever mass has the characteristic quality of fluidity. It follows also as a consequence of this mode of treating the subject, that a mass of fluid may with propriety be considered as a machine for the transmission of pressure equally in all directions; and so long as the volume of the fluid remains unaltered, that is to say, so long as its component particles are related together by the same system of internal forces, the fluid will constitute the same machine, to whatever forces or pressures it may be subjected*. Such

* It will follow also as a consequence of these principles, that the equation of virtual velocities which applies to other machines holds in a mass of fluid in equilibrium and subject to pressure. See Webster's *Theory of Fluids*, Art. 8.

being the law or property of fluids, acted on by external forces or subjected to pressure, we shall proceed to consider in the next Chapter some of the consequences when the only impressed force is the force of gravity.

28 *a*. *Armstrong's Hydraulic Cranes.* The hydraulic crane recently introduced by Mr Wm. Armstrong at the London and Liverpool Docks and other places affords an instructive illustration of the equal transmission of pressure throughout a fluid mass, and of the equality of pressure which exists at every part of a fluid contained in any system or series of vessels in communication with each other. In cranes for raising or lowering large weights, as hogsheads of sugar out of or into the hold of a vessel, it is necessary to have the means of stopping the motion of the weight raised at any moment; this is done by cutting off the admission, or emission of the fluid constituting the machine which connects the power and the weight. The power in such cases is the pressure produced by the weight of a column of fluid, or by pumping air into a close vessel containing a liquid, or by raising a weight by water; this pressure is transmitted through the whole liquid contained in the system of pipes, to the pistons whose motion actuates the wheel-work and chains by which the weight is raised or lowered, according to the side of the piston on which the water acts.

The pressure at the piston and by which the weight is raised or lowered, will vary exactly with the pressure at the source of power, and when the weight is at rest will be the same at each equal portion of surface, as on each square inch, at the piston and at the source of power; that is, there will be equilibrium throughout the entire system*.

* See hereafter in Chapter X. as to *Water-Pressure Engines*.

CHAPTER III.

ON THE EQUILIBRIUM OF FLUIDS SUBJECT TO GRAVITY.

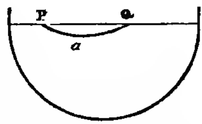
29. In the preceding Chapter the equilibrium of a fluid mass considered as a collection of particles devoid of weight, but subject to an impressed force, has been treated of. Though this imaginary case serves well to exhibit the property of a fluid considered as a machine, it does not fully represent the cases which occur in nature. But having thus ascertained the peculiar property of a fluid subject to pressure, we may proceed to treat of the laws of the equilibrium of a fluid mass, every particle of which is acted on by gravity. In this inquiry we shall have to treat of the form of surface and the pressure at any point. In the preceding case, the form of the surface depended on the containing vessel, since every particle was equally pressed against the sides; but in the following, the surface is free to assume the form which may be due to the impressed forces.

30. Prop. *When a fluid is at rest, any portion of it may be conceived to be separated from the surrounding fluid, and enclosed in a rigid surface, without altering the forces exerted.*

Since all the particles of the fluid are in equilibrium, we may suppose any of them to be rigidly connected, and the forces will remain as before; for the change to rigidity introduces no active force, but only those forces which would resist a tendency to change of position, if there were any such tendency; but there is not, therefore the forces of rigidity are not called into action.

Let the film of particles which surrounds any portion of the fluid become rigid; then there is no change in the forces exerted, and the pressure of the fluid on each side of the film will be resisted by the rigidity; and the particles of the fluid on the two sides of the film cease to exert forces on one another, these forces being supplied by the rigidity. Thus any portion may be conceived to be separated from the surrounding fluid, and the action of this portion on any particular point may be considered independently of the rest of the fluid.

31. Prop. *The pressure at every point of the surface of a fluid at rest is the same.*



Let P and Q be any two points in the surface of a fluid at rest, then the pressure is the same at these points. For let any portion of the fluid, as PaQ, be conceived to be detached from the rest, and enclosed in a rigid tube (Art. 30). Then the pressure at P is accurately transmitted along the line of particles PaQ to Q (Art. 23), and if it be not counteracted by an equal pressure at this point, the equilibrium of the fluid will be disturbed, which is contrary to the hypothesis. Conversely, when the pressure at all points of the surface of a fluid is the same, the fluid is in equilibrium.

Cor. Hence the pressure on all equal portions of the surface of a fluid at rest is the same.

32. Prop. *The surface of a fluid of small extent at rest is horizontal.*

This may be shewn to be the necessary consequence of the action of gravity on the particles of a fluid mass, and may be verified by experiment.

Gravity at the earth's surface is a uniform force*, that is,

* Whewell's *Elementary Mechanics*, sixth edition, Art. 60.

it produces the same effects on every particle of matter at the same distance from the centre of the earth. Now for portions of the surface of small extent every point at the same distance from the centre may be considered to be in the same horizontal plane. Hence if the surface of a fluid is horizontal, all the particles at its surface are equally acted upon by gravity, and the pressure at every point of the surface of the fluid is the same. But the pressure at all points of the surface of a fluid being the same the fluid is in equilibrium. A small extent consequently of the surface of a fluid at rest, and subject to gravity, is horizontal.

Let a variety of vessels of very different forms and magnitude, as for instance, cylindrical or spiral tubes, and conical or spherical vessels, stand near and communicate with each other by means of a tube passing through their lower parts and opening into each of them. Now it is a well-known fact, that if water or any heavy fluid be poured into such a system of vessels, the fluid will stand at the same height in all of them, and if a line be drawn touching the surface of the fluid in any two of them, the same line will touch the surfaces of the fluid in all the rest. This line appears to be parallel to the horizon, and all the surfaces appear to be in the same horizontal plane. Also if the surface of the fluid in a large tub be observed, it will appear to be in the same horizontal plane; and if the preceding remarks be correct, it must be so. For any two or more portions of the surface may be considered as the surfaces of fluid enclosed in tubes (Art. 30), and the surfaces in these tubes being horizontal, the whole surface is horizontal also.

Thus the horizontality of the surface of still water of small extent may be inferred, and conversely, the horizontality of a fluid at rest is the test whereby the horizontality of any other surface is determined.

When however the surface is of very small extent there are some remarkable exceptions to the preceding law, which will be treated of in the Chapter on Capillary Attraction.

33. Prop. *The surface of a fluid at rest is a level surface.*

In common language the term level is the same as horizontal, and this may be taken as its true meaning, when a small extent of surface is spoken of, but it will be seen to have a very different meaning when applied to the surface of fluids of great extent, as the sea, or large lakes.

The earth's figure may be considered as spherical, for the slight deviation from this form may be disregarded in the present inquiry. Now the inequalities on the surface of the terrestrial part of the earth, insignificant though they be when compared with the whole mass of the earth, do not exist at all on the surface of still water, as on the surface of a calm sea. But it is known by observation that the surface of the sea is uniformly curved, and would, if continued uninterruptedly in every direction, present a spherical surface. This is the observed fact, and the same conclusion may be arrived at in the following manner.

Gravity at the earth's surface is a uniform force, and acts in a direction perpendicular to that surface, that is, in the direction of the plumb-line. It acts equally on all bodies at equal distances from the centre of the earth, and unequally on bodies at different distances, exerting a greater action on those at a less distance than on those at a greater. Hence if the surface of still water have all its points at the same distance from the centre of the earth, every particle at its surface is equally acted on by gravity. But if any two points on its surface be at different distances from the centre, the particles at the points will be acted on unequally, and the pressure at different points of the surface being unequal,

the fluid will not be at rest (Art. 31). Hence it is evident, that under the action of a force, such as gravity is known to be, a fluid mass must settle down into a spherical form, and the fluid will not be at rest unless it is in this form; and this agrees, as was just stated, with the observed fact. This spherical surface is called a level surface; and if the whole globe were covered with water, there would have been but one surface, and therefore but one level. But the fact is, that there are many different surfaces, and though theory shews that they all ought to be, and observation shews that they all are, spherical surfaces, they have not the same radius, but are at very different distances from the centre. This is expressed by saying that they are all level surfaces, but not all on the same level; and one level is said to be above or below another level, according as it is at a greater or a less distance from the centre of the earth.

From the preceding it appears that a level surface may be defined as the surface which a fluid, as water, assumes when acted on by gravity*

Cor. The pressure must be the same at every point of each of these levels; for every point is similarly situated with respect to gravity. The preceding may be expressed by saying, that all levels are surfaces of equal pressure and consequently of equal density.

34. Prop. *Fluids rise to the same level throughout a system of communicating vessels.*

Since the surface of a fluid at rest is a level surface, it follows that a fluid in a system of vessels in free com-

* The general proposition respecting the direction of the forces acting on the surface of a fluid in equilibrium and the form of the surface, is that the resultant of all the forces acting at any one point will be perpendicular to the surface at that point. (*Theory of Fluids*, Art. 16.) Thus in the case of the figure of the earth, which is more curved at the equator than at the poles, owing to the rotation of the earth about its axis, the centrifugal force at the equator is combined with gravity, and it is the resultant of these forces to which the surface is perpendicular.

munication with each other cannot be at rest unless the surface of the fluid in these different vessels is on the same level. Hence when water is supplied to any part of such a system of vessels, it will immediately rise in some and sink in others until this level is obtained, when all motion will cease, and the fluid will be at rest. When the vessels are situated near each other, we have seen (Art. 32) that the surface in all of them is horizontal, for then the horizontal and level surface may be considered as coincident; and when the vessels are at great distances, as many miles apart, the surfaces will be on the same level.

35. *Supply of towns with water.* The practical application of the preceding proposition is exhibited in the methods adopted for the supply of towns with water. The ancient method of supplying towns with water was by means of aqueducts. These stupendous and costly constructions could scarce have been rendered necessary from ignorance of the principle that fluids rise to a common level, but probably from the practical difficulty of making pipes water-tight at the joint; they served moreover the purposes of architectural embellishment.

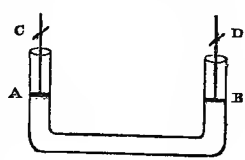
But aqueducts, for the purpose of supplying towns with water, are now almost entirely superseded by long pipes laid under ground and above ground, turned and twisted in any direction to suit the locality of the places to be supplied with water. For this purpose a reservoir is selected in some situation more elevated than the places to which the water is to be supplied. This reservoir is fed from natural sources, as surface drainage springs, and rivers, or water is raised up into it by suitable apparatus. Pipes are conveyed from it in the required directions, and the water may be "laid on," that is, supplied at the tops of houses at any distance, provided the tops of the houses be not above the

level of the water in the reservoir. The determination of the points at which the water can be supplied, that is, of the points on the same level with the water in the reservoir, is treated of in the following articles.

The New River Water Works, which supply part of London with water, present an exemplification both of the ancient and modern methods. The water is conveyed for many miles in an artificial channel or cut, and by aqueducts across the valleys, from Ware to the open reservoir at the New River Head at Islington. This reservoir is sufficiently elevated to supply the lower parts of the district with water. To supply the rest, water is raised by a powerful Steam Engine to a more elevated reservoir situated on Pentonville Hill; and the elevation of this point is sufficient to supply nearly all the rest of the district. The other places which are above the level of the water in this reservoir, are supplied by means of a forcing pump and air vessel, the construction and working of which will be explained in a subsequent Chapter.

36. *The Level.* It has been seen (Art. 33) that a level surface is one every point of which may be considered as situated at the same distance from the centre of the earth. And the operation of levelling consists in determining the relation between the levels in which different objects are situated; and this may be determined by instruments whose construction depends on the property that the surface of a small extent of fluid when at rest is horizontal (Art. 32). An instrument for this purpose is termed a Level.

A Level in its simplest form consists of a tube with its two ends turned up and open, and nearly filled with water. Upon the surfaces of the fluid in these two ends are two floats A and B, carrying uprights with sights C, D, at equal distances



above the floats. These sights may be small pieces of wood or squares with cross wires intersecting at right angles at the centre of the square. Now if when this instrument is placed on any surface, or held in the hand, the sight at *C* covers the sight at *D*, the points *C* and *D* are in the same horizontal line. For the floats at *A* and *B* are horizontal (Art. 39), and the line joining *C* and *D* is parallel to the line joining these, since the distance *CA* equals the distance *DB*, and therefore *C* and *D* are in the same horizontal line. And if the sights cover any third point, these three points are in the same horizontal line. It is specially to be observed, with respect to this instrument, that the position in which it is held is immaterial, since the floats must be horizontal when the fluid is at rest.

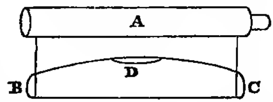
37. *The Spirit Level.* The common Level has some practical disadvantages, and the instrument generally used is termed the Spirit Level, because alcohol, which does not readily freeze, is the fluid used. A glass tube having its lower surface plane and its upper slightly convex, is nearly filled with the fluid, and, a small air bubble being left in, hermetically sealed.

Now air being a lighter fluid than the spirits of wine, the air bubble will always occupy the highest part of the tube. Consequently when the lower surface of the tube is horizontal, the surface of the fluid will be parallel to it, and the air bubble will occupy what is then the highest part of the level, that is, it will be just under the middle part of the convex surface. Again, if the surface on which it is laid, or the position in which it is held, incline in any direction to the horizontal, the surface of the fluid is not parallel to it, and the air bubble will move towards the higher end.

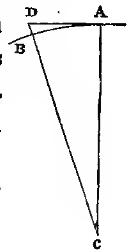
Thus the position of the air bubble, with reference to

the middle of the upper surface of the tube, may determine the actual position of any surface with respect to the horizontal.

For the practical purposes of levelling, a telescope A is attached to the level BC having its axis parallel to the lower surface of the tube. So that when the level is horizontal, all the points seen through the telescope are in the same horizontal line, and the air bubble D is under the middle of the upper surface.



38. *The Depression.* Let the Level with the attached telescope be fixed in a horizontal position at any point A, on the surface of the earth. Then all the points seen through the telescope will be in the horizontal line AD. But the point which is required to be determined is some point B, at a given distance AB from A, on the same level as A.



Let C be the centre of the earth, then joining CB and producing it to D, the point B, which is required, is below the horizontal line AD by the distance DB.

This distance DB is termed the depression, and may be calculated as follows*.

* Or by Trigonometry as follows: let θ be the angle ACB; then $\lambda = r\theta$, and the depression being the difference between the secant and radius, we have the depression $= r(\sec\theta - 1) = r\left(\frac{1}{\cos\theta} - 1\right)$. But $\frac{1}{\cos\theta}$ $= \frac{1}{1 - \frac{1}{1.2}\theta^2 + \&c.} = 1 + \frac{\theta^2}{2}$ nearly (when $\theta = \frac{\lambda}{r}$ is small), and the depression $= \frac{\lambda^2}{2r}$, as in the text.

Let λ be the distance AB in miles, and x the depression, and r the radius of the earth; then

$$CD^2 = CA^2 + AD^2 \text{ or } (r+x)^2 = r^2 + AD^2,$$

$$\text{whence } 2rx + x^2 = AD^2 \text{ or } x + \frac{x^2}{2r} = \frac{AD^2}{2r}.$$

Now for ordinary distances, as a few miles on the earth's surface, the term $\frac{x^2}{2r}$ may be omitted, and AD assumed equal to AB without appreciable error;

$$\therefore \text{ the depression} = \frac{\lambda^2}{2r}.$$

In this equation r may be assumed equal to 3900 miles, and λ will be known, either from actual admeasurement, or having been determined by requisite observations.

To render the preceding formula for the depression useful, an approximate numerical value must be obtained. For this purpose multiply the right-hand side by the number of feet in a mile;

$$\therefore \text{ the depression in feet} = \frac{\lambda^2}{2 \times 3900} \times 5280$$

$$= \frac{44}{65}\lambda^2 = \frac{2}{3}\lambda^2, \text{ nearly.}$$

Hence if the point D be known by a staff being set up, or if advantage be taken of a church steeple, or any other convenient object, the point B on the same level with A will be situated below D by a distance very nearly equal to $\frac{2}{3}\lambda^2$.

The value of the depression may also be expressed in inches. For this purpose, multiply the right-hand side by the number of inches in a foot;

$$\therefore \text{ the depression in inches} = \tfrac{2}{3}\lambda^2 \times 12 = 8\lambda^2.$$

It must be remembered that λ is the distance expressed in miles.

When the objects are several miles apart, the preceding formula must be corrected for the atmospheric refraction. The effect of the refraction is to render an object viewed from a distance apparently more elevated than it really is; hence the preceding value of the depression is too large by about one-tenth. The more accurate value of the depression corrected for the refraction is $\frac{4}{7}\lambda^2$; but the amount of terrestrial refraction is so variable, depending as it does upon the temperature and density of the air at the place of observation, that the above formula gives approximate results only, not differing however in general so much as a tenth from the true value of the quantities to be determined.

39. *Examples.* The following examples will shew the use which may be made of the formula of the preceding article for the approximate solution of practical questions.

Ex. 1. How far can a person six feet high see on a level plain?

$$\text{Then } 6 = \tfrac{2}{3}\lambda^2; \quad \therefore \lambda^2 = 9, \text{ or } \lambda = 3.$$

He can see, therefore, three miles all around him.

Ex. 2. A light of known elevation on a cliff is just visible from the mast-head of a vessel. Required the distance of the vessel from the cliff.

Let the light be 100, and the mast-head 70 feet above the level of the sea. Then there will be a point on the surface of the sea between the cliff and the vessel, which point is on the horizontal line between the light and the mast-head, and the distance of the vessel from the cliff may be taken to be the sum of the distance of the cliff from that intermediate point, and of the distance of that intermediate point from the vessel. Hence, applying the formula just obtained, we have

$$\text{The distance} = \sqrt{\frac{3}{2}\times 100} + \sqrt{\frac{3}{2}\times 70}$$

$$= 12 + 10, \text{ or } 22 \text{ nearly.}$$

The vessel is therefore not less than 22 miles distant from the cliff.

40. *Levelling.* The operation of Levelling consists in finding a series of points on the same level, and in determining the heights and depths of places with respect to each other, or some standard level, as the mean level of the sea. We have seen, in the two preceding articles, the application which may be made of the formula in important practical questions; and in making canals, roads and railways, the skill of the engineer is shewn in selecting that line of country which is as much as possible on a natural level.

Thus in conducting a canal across a country, the points in the same level must be determined, so that if it be possible no locks may be required, but the water may rest running neither way.

It will often happen that a canal, road or drain, is to cross a deep valley, and the points on the opposite sides of this valley which are on the same level must be accurately determined. This may be done approximately by the preceding proposition, for the point in the same horizontal line on the opposite side of the valley will be determined by setting the level horizontally, and looking through the telescope, and the distance of the point on the same level below it will be given by the value of the depression just obtained. But this method of determining the required point is not, for the reasons already mentioned (Art. 38), sufficiently accurate, and recourse must be had to other observations.

The places to which water can be supplied in towns (Art. 35), will be determined in the same way. In practice,

however, it is most frequently required to conduct a canal or drain from one point to another point. Here the extreme points are given, and it is required to find a number of intermediate points on levels neither above the level of the higher, nor below the level of the lower point; the magnitude of the depression, or, the quantity of fall between these two points is readily determined, by setting the level at the two extremities, and observing how much one extremity is above one or more intermediate points, and how much the other is below it. Hence, knowing the whole fall and the whole distance, the quantity of fall for any portion of that distance is determined by a simple proportion*.

* The reader is referred to the *Treatise on Mathematical Instruments*, by F. W. Simms, to Bourn's *Principles and Practice of Surveying*, to Bruff's *Engineering Field Work*, to F. W. Simms' *Treatise on Levelling*, (4th ed. Weale), and to Baker on *Land and Engineering Surveying*, (Weale's series of works), for information on Levelling as practically performed.

CHAPTER IV.

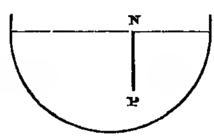
ON THE PRESSURE OF A FLUID SUBJECT TO GRAVITY.

41. In treating of the equilibrium of a fluid subject to pressure (Chap. II.), and supposed devoid of weight, it appeared to be a necessary consequence of the application of pressure to a fluid mass, that the pressure would be the same at every point throughout the fluid mass, and on every equal portion of the containing vessel. But the pressure which results from the impressed force of gravity varies in magnitude according to the position of the point in the containing vessel.

In the preceding Chapter (Art. 31) the condition for the equilibrium was seen to be, that the pressure should be the same at every point of the surface. We have now to investigate the amount of pressure at any point in the interior.

42. Prop. *The pressure at any point is proportional to the depth of the point below the surface of the fluid.*

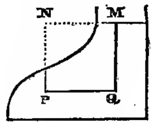
(1) Let the point P be situated directly below the surface of the fluid. Now since every part of the fluid is at rest, the particles in the line PN drawn perpendicular to the surface of the fluid may be conceived to be separated from the surrounding fluid, and enclosed in a rigid surface by a film of the surrounding fluid becoming rigid (Art. 30).



Now every particle of the fluid in PN has weight (Art. 8), and the weight of each particle is transmitted along the line NP to the

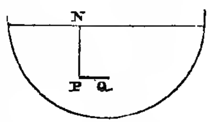
point P (Art. 23), so that the pressure at this point is the aggregate weight of the particles in this line. Hence the pressure at P is proportional to the number of particles in PN, that is, to the depth of the point below the surface of the fluid.

(2) Let the point P be situated not directly below the surface of the fluid. Let PQ be drawn parallel, and QM perpendicular to the surface of the fluid; and let the particles in these lines be separated from the surrounding fluid by a rigid surface. Then the pressure is the same at all points along the line PQ (Cor. Art. 33), and the pressure at P is equal to the pressure transmitted along the line PQ from Q, which by the first case is proportional to the depth of Q; and $PN = QM$. Therefore the pressure at P is proportional to the depth of that point below the surface of the fluid.



43. Prop. *To find the pressure on a portion of a plane immersed in a fluid.*

(1) Let the plane be parallel to the surface of the fluid, and let the portion of it the pressure on which is required be equal to the unit of area.

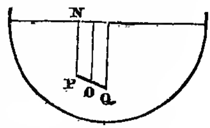


Let PQ be this portion at the depth PN. Now the column whose base is PQ may be conceived to be separated from the surrounding fluid, and enclosed in a tube. Then the pressure on PQ is entirely independent of the rest of the fluid, and is equal to the weight of the column.

The weight of any column is the aggregate weight of the particles composing it. And the weight of each unit of bulk being represented by $g\rho$ (Art. 13), and z being the depth of the plane, that is, the number of units of height

in the line PN, the weight of the whole column is $g\rho z$. Hence (Art. 25) $p = g\rho z$.

(2) Let the plane be inclined to the surface of the fluid, and let PQ equal to the unit of area be the portion on which the pressure is required.



Let z be the depth of the middle point O of this plane, and $z_{\prime}$, z', the depths of the extremities P and Q. Now since the pressure is proportional to the depth (Art. 42), the pressure on PQ is evidently greater than the pressure on an equal area passing parallel to the surface of the fluid through P, that is, at the uniform depth $z_{\prime}$, and less than the pressure on an equal area through Q at the uniform depth z'. Hence by the preceding case

$$p \text{ is greater than } g\rho z_{\prime},$$
$$\text{and} \quad \dots \text{ less } \quad \dots \ g\rho z'.$$
$$\text{Let } z_{\prime} = z - h\,; \quad \therefore\ z' = z + h\,;$$
$$\therefore\ p \text{ is} > g\rho z - g\rho h,$$
$$< g\rho z + g\rho h\,;$$

and this is true whatever be the value of h. Consequently

$$p = g\rho z.$$

For if not, it must be greater or less than it. Let it be greater than it, and suppose that $p = g\rho z + a$. Then

$$g\rho z + a < g\rho z + g\rho h\,;$$
$$\therefore\ a < g\rho h,$$

which is impossible, since h may have any value whatever, and differ consequently from zero by a quantity less than any which can be assigned.

Next, let it be less than it, and suppose that $p = g\rho z - a$,

$$\therefore\ g\rho z - a > g\rho z - g\rho h,$$
$$a < g\rho h\,;$$

which is as before impossible. Hence

$$p = g\rho z.$$

COR. Hence if any portion of a surface be represented by A, where A is either the number of units of area contained in the given surface, or that portion of the unit of area which the given surface is, the pressure on that surface if at the depth z is $pA = g\rho zA$. In the preceding reasoning the quantity ρ which expresses the density is supposed to be invariable, which will practically be the case for small columns of the inelastic fluids (Art. 19).

In general the pressure of the atmosphere on the portion of the surface of the fluid, which is equal to the unit of the area, must be added to the weight of the fluid column for the true value of p.

44. PROP. *To find the pressure of a fluid on any surface immersed in it, or on the surface of the containing vessel.*

Let the surface, whether plane or not, be divided into small portions, each of which may be considered plane.

Let S be the whole surface; k, k', k'', &c. the portions into which it is divided, and z, z', z'', &c. their mean depths. Then the pressure of the fluid on these is $g\rho zk$, $g\rho z'k'$, $g\rho z''k''$,... (Cor. Art. 43) respectively; therefore the pressure on the surface

$$= g\rho zk + g\rho z'k' + g\rho z''k'' + \&c.$$

Let Z be the depth of the centre of gravity of the whole surface below the surface of the fluid; then (since the mean depths of the small portions into which the surface is divided are the depths of the centres of gravity of these portions) by the property of the centre of gravity*,

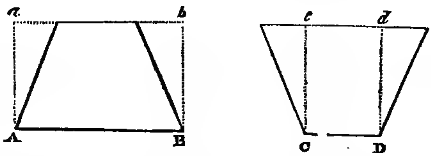
$$g\rho zk + g\rho z'k' + g\rho z''k'' + \ldots = g\rho S \,.\, Z.$$

But $g\rho ZS$ is the weight of a column of fluid whose height is Z, and base S. Whence it appears that *the pressure of*

* The property of the centre of gravity here alluded to is, "That the effect of any system to produce equilibrium is the same as if it were collected at its centre of gravity." See Whewell's *Elementary Mechanics*, sixth edition, Art. 43. Potter's *Mechanics*, Chap. v.

a fluid on any surface is the weight of a column of the fluid whose base is equal to the area of the surface pressed, and whose height is equal to the depth of the centre of gravity of the surface below the surface of the fluid.

45. *Pressure on the bottoms of vessels.* If the sides of a vessel be vertical, and its base parallel to the surface of

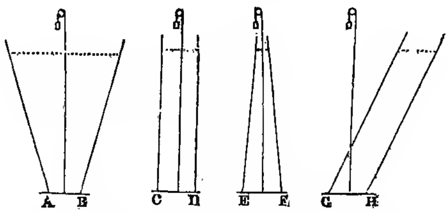


the fluid, the pressure of the fluid on its base is the whole weight of the fluid contained (Art. 44). But if the sides of the vessel be inclined, as represented in the figure, the pressure on the base is in one case greater, and in the other less than the weight of the fluid contained. For the pressure on the base AB is the weight of the column $AabB$, and the pressure on CD is the weight of the column $CcdD$, the former of which is greater, and the latter less than the weight of the fluid contained; these being the columns whose bases are the surfaces pressed, and heights the depths of their centres of gravity (Art. 44). If the vessel be narrower at the top than at the bottom, and therefore the pressure on the base AB be greater than the weight of the fluid, the additional pressure arises from the re-action of the sides. And if water be poured into a hollow cone resting on a horizontal plane, the water will raise the cone unless the cone be very heavy. If the vessel be wider at the top than at the bottom, the remaining weight is sustained by the sides.

A distinction must be made between the pressure which the base AB sustains from the fluid alone, and that which it sustains in supporting the vessel and the fluid contained.

For if the bottom were detached from the vessel, the only force requisite to keep the bottom close to the sides, and to prevent the escape of the fluid, is a pressure upwards equal to the weight of the column of the fluid. Whereas the pressure requisite to support the vessel, is the weight of the vessel and of the fluid contained. And when the vessel is narrowest at its base, it requires more force to support the vessel than to keep the base from falling, and when widest (*cæteris paribus*) it requires less.

46. *Illustrations.* The conelusions arrived at in the preceding articles may be illustrated by experiment in the following manner. Let four vessels, as represented, have the



apertures at their bottoms all of the same area and closed by plates AB, CD, EF, and GH of equal weight. Let these plates be kept in their places by the tension of strings acted on by equal weights, the weight attached to the strings will serve as the measure of the pressure of the water on the plates or bases of the vessels respectively. If water be ponred into the vessels, the plates closing the orifices at the bottom will descend and allow the water to escape when the pressure on the plate becomes greater than the weight attached to the string, and it will be found that when the plates descend, the water will be at the same height in all the vessels. The same result would follow, whatever the shape of the vessels, and although one may hold many hundred times the contents of the other.

47. *Pressure on the sides of vessels.* The preceding proposition (Art. 44) also enables us to determine the pressure against the side of a vessel, in all cases in which the depth of the centre of gravity of that side, or of the portion in contact with the fluid below the surface, is known. The determination of this point for vessels having plane sides and of a rectangular or triangular form, or in conical and cylindrical vessels, presents few difficulties, but in most other cases recourse must be had to the assistance of analysis*.

Ex. The pressure against the side of a cubical vessel full of water is half the weight of the fluid. For the pressure on the bottom is $g\rho Sz$, or the whole weight of the fluid, and on a side is $g\rho S.\frac{z}{2} = \frac{1}{2} g\rho zS = \frac{1}{2}$ the weight of the fluid.

48. *Centre of pressure.* The amount of the pressure on the bottom or sides of vessels containing or in contact with a fluid, or on a plane immersed in a fluid, and the formula for its determination, have been treated of in the preceding articles.

The amount of that pressure having been determined, the question arises, whether that pressure can be represented or counterbalanced by a single contravalent force, and if so, at what point such single force should be applied. The point in any surface at which a single force must be applied, so as to keep in equilibrium or counterbalance the fluid pressure to which the surface is subject, is called the centre of pressure of the surface.

If the bottom of the vessel containing the fluid or the plane immersed in the fluid be horizontal, the pressure on every point is the same; and the fluid pressures or the forces being parallel, the resultant of these forces will pass through

* See *Theory of Fluids*, Art. 46.

the centre of gravity of the bottom or the plane, and a single force applied at that point will sustain or counterbalance the pressures to which the bottom or plane is subject. In this case then the centre of pressure coincides with the centre of gravity.

But if the bottom of the vessel or the plane immersed be not horizontal or parallel to the surface of the fluid, but inclined to it, or if as in the case of the side of the vessel the pressures though parallel to each other are not all equal, being proportional to the depth of the point below the surface of the fluid (Art. 42), the resultant of these pressures will not pass through the centre of gravity, but through a point below it, and the point at which the single contravalent force is to be applied to counterbalance the fluid pressures is below the centre of gravity of the inclined surface or plane or side of the vessel.

If the bottom, side, or part, of the containing vessel be supposed moveable, it will be kept at rest by a single force or pressure, equal to the sum of the fluid pressures to which the bottom, side, or part, of the containing vessel is subject, and applied in the opposite direction to those pressures.

The position of the centre of pressure of any surface may be readily derived from the principles of statical equilibrium and the pressure of fluids; but as the formulæ requisite for the determination of this point in all cases require some knowledge of the differential and integral calculus, we shall confine ourselves, in the present treatise, to pointing out the position of this point as determined by analysis, in some of the cases which ordinarily occur in practice*.

49. *Illustrations.* It appears from the formulæ referred to in the last article, that the centre of pressure of any four-

* See Webster's *Theory of Fluids*, Art. 49, for the general formula for determining the centre of pressure.

sided rectangular figure placed vertically is at the depth of two-thirds of the height of the figure below the surface of the fluid, the upper edge of the figure coinciding with the surface of the fluid. Hence if a rectangular flood-gate turn about a horizontal axis, and be acted on by a force, as the tension of a chain sustaining a weight, applied at a point in the gate, one-third of the proper depth of the fluid from the bottom of the gate, the gate may be kept closed; but if the pressure against the gate be increased, as by the water rising higher than its proper level, the gate will open and allow the water to escape, and close again when the water has subsided to its proper level.

The staves of a barrel containing liquid may be kept in their places by a single hoop placed at a distance of two-thirds from the top of the barrel, or one-third from the bottom, the barrel standing on one end. For each of the staves would be kept at rest by a single force applied at a point one-third from its bottom, that being the position of the centre of pressure, and a hoop placed at this point will produce the same effect.

An accurate knowledge of the position of the centre of pressure is of the greatest importance in the construction of all works which will have to sustain the pressure of water, as embankments, reservoirs, flood-gates, and tanks, so as to ensure the proper disposition of materials, and sufficient strength in the parts where the pressures may be securely counterbalanced.

50. Prop. *At a great depth below the surface of the fluid the centre of pressure coincides with the centre of gravity.*

It will be seen on examining the formulæ for determining the centre of pressure, and the centre of gravity of a surface, that when the surface is sunk to a great depth, the

formulæ are the same; thus shewing that in this extreme case the two centres coincide with each other*. The reason of this will be obvious, the fluid pressures at different points are parallel to each other and to the direction of the force of gravity; but the pressures at any two points differ from each other in proportion to the difference of the depths of the two points below the surface; when this depth is considerable, the differences between the depths of the several points of the same plane are inconsiderable, as compared with the whole depth, and consequently the differences may be disregarded, or the pressures may be considered equal as well as parallel, the resultant therefore of those pressures will pass through the centre of gravity.

The flying of a kite affords a good illustration of the above proposition; for the kite is sustained by a single force, the tension of the string, the pressure of the air being counterbalanced thereby; now the air is a fluid of great depth, and it will be found that the proper point for attaching the string is at the centre of gravity of the kite.

Upon the same principles it will follow, that in the case of a vessel containing an elastic fluid, the weight of which is inconsiderable, the centres of pressure of the sides will coincide with their centres of gravity.

51. Prop. *The resultant of the fluid pressures acting at any point on the surface is perpendicular to the surface at that point.*

The amount of the pressure of a fluid on any surface, as the side of a containing vessel, has been treated of in the preceding articles, and it appears that the whole pressure may be counterbalanced by a single force acting at one point.

Now if the fluid pressures may be counterbalanced or retained by a single force, the resultant of those pressures, as

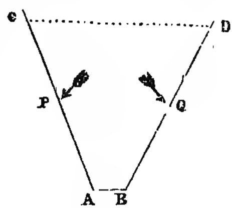
* *Theory of Fluids*, Art. 59.

well as the single contravalent force, must act opposite to each other, and in a direction perpendicular to the surface at the point of their application. This conclusion, which may be strictly derived from the principles of statical equilibrium *, will be evident from the following considerations. A fluid has been defined to be a collection of particles capable of acting on and moving amongst each other in every direction, without an appreciable amount of friction (Art. 4). Now it is involved in our conception of the absence of friction, that the direction of the force should be perpendicular to the surface at the point of action, and consequently that the action of fluid pressures at any point of a surface is perpendicular to the surface at that point.

52. PROP. *The horizontal pressures of a fluid on the sides of a containing vessel are in equilibrium with each other.*

The resultant of the fluid pressures at any point of a containing surface being perpendicular to the surface at that point, it will follow from the principles of statical equilibrium, that the horizontal pressures of the fluid are in equilibrium with or counterbalance each other †.

Let a tall vessel with a small base AB, and inclined sides AC and BD, be filled with water to the level CD. Then the pressures on the sides AC and BD may be represented by the forces P and Q, acting perpendicularly to the sides at their respective centres of pressure. Now there is no tendency in a vessel so filled to overturn or move laterally; but if the resistance of one of the sides be

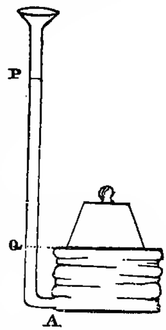


* See *Theory of Fluids*, Art. 16. † Ib. Art. 55.

destroyed, as by the removal of a portion of the surface, as opening a hole, the vessel will overturn.

An application of this principle will be seen hereafter in Barker's Mill, and it has been proposed by Bernouilli and others to impel vessels forward by re-action, water being allowed to flow out at the stern. The same principle is exhibited in the machines which derive their motion from the action of steam issuing out of an arm, and in the force by which a rocket is impelled forward.

53. *Hydrostatic Bellows.* Two pieces of board are united together by leather or any flexible substance which is water-proof, so as to form what is termed a pair of bellows. A small vertical tube communicates with the interior of the bellows; and if water be poured into the bellows down this small tube it will enter and raise the upper board, even if it is loaded with a heavy weight, as represented in the figure. And the upper board will continue to rise until either the bellows or tube is full.



When an equilibrium exists between the water in the tube and bellows, let the water in the tube stand at a height PQ above the surface of the water in the bellows. Then PQ is the column of fluid which supports the whole weight raised; for the fluid in QA is in equilibrium with the fluid in the bellows, and therefore the pressure at the surface of the fluid in the bellows is in equilibrium with PQ.

Let W be the whole weight raised; namely, the weight of the upper board, of the leather, and of the load.

Let K be the area of the upper board, and k of a section of the small tube.

The pressure exerted on the surface of the fluid at Q, by

he superincumbent column PQ, is transmitted to every equal portion of the surface of the upper board (Art. 23).

The upward pressure on the upper board is pK, if p be the unit of pressure, that is, the pressure on a unit of surface which is transmitted from the surface at Q. And this is the pressure which sustains the weight raised, or W;

$$\therefore W = pK.$$

But the whole pressure at $Q = pk$;

$$\therefore W : \text{pressure at } Q :: pK : pk$$
$$:: K : k;$$

$$\therefore \text{the weight raised} = \frac{K}{k}.(\text{pressure at } Q)$$
$$= \frac{K}{k}.(\text{weight of the column } PQ).$$

Hence the weight which can be raised will be increased, by the increase of the area of the upper board, by the diminution of the section of the tube, and by increasing the pressure of the small column, that is, by increasing the altitude of the tube.

Now theoretically the ratio $\frac{K}{k}$ may become infinite, and the altitude of the tube may also be increased indefinitely; hence it is evident that any quantity of fluid however small may be so employed as to sustain any weight however large.

This last conclusion is in substance the same as the proposition already (Art. 27) established, that forces having any assignable ratio to each other may, by the transmission of pressure in a fluid, be in equilibrium; for the pressure at Q in the small tube or tube of section k is in equilibrium with the pressure resulting from the larger weight on the board of an area K.

54. *Illustrations.* The conclusions arrived at in the preceding articles may be readily verified and illustrated. For instance, if a small pipe be inserted into a barrel containing water or other liquid, and water or other liquid be poured into the pipe, the barrel may easily be burst; the length of pipe and height to which it must be filled will depend on the strength of the barrel. If a heavier liquid, as mercury, be used to produce the pressure on the surface of the liquid in the barrel, a column of about one thirteenth the height of the column required, if water be used, would be sufficient. And the same effects may readily be produced by a pressure exerted on the water in the tube by a piston or otherwise.

It is highly probable that many of the phenomena of dislocation of large masses of earth and of other convulsions of nature are due solely to the pressure of water. For suppose water to collect for a considerable extent underneath a water-tight stratum, or in some large cavern in the interior of a mountain, and that this water is subject to the pressure of a column of several hundred feet in height; the pressure underneath the water-tight stratum or in the interior of the mountain may thus become greater than the materials can bear, and a violent disruption may suddenly take place.

The same powerful effect may be produced by the transmitted pressure of elastic fluids; thus if a person were to stand on the large board of the hydrostatic bellows he might raise himself by blowing with his mouth down a small tube of caoutchouc or other air-tight substance. In such a case the transmitted pressure would be determined by the power of the lungs of the operator, and not by the height of the column in the small tube.

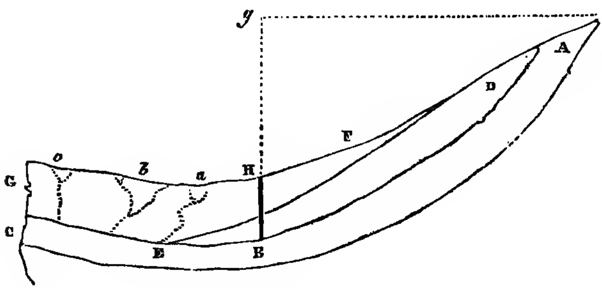
55. *Hydrostatic Paradox.* The conclusions contained in the preceding articles, though the necessary conse-

quences of the characteristic of a fluid, namely, the equal transmission of pressure in all directions, may be expressed in language apparently so contrary to our natural conceptions as to appear paradoxical; and the above truths have been presented in such a form as to acquire the designation of the *hydrostatic paradox;* but when the sense in which the terms are used is understood, the proposition thereby enunciated is most intelligible and almost self-evident. The truth called the hydrostatic paradox has frequently been propounded in the following terms: "That the pressure on the bottom of vessels containing fluid does not depend upon the quantity of fluid." Now this proposition is not to be taken literally in the strictest sense of the terms, but in a technical and conventional sense, and it may be explained as follows: To increase the pressure on the bottoms of vessels it is not necessary to increase the quantity of the fluid, but only the altitude of the column or the form of the containing vessel; thus the pressure on the bottom depends on the manner in which the fluid is disposed, rather than on the quantity. If a given quantity of water be spread in a thin sheet over a large surface, the pressure at any one point of that surface will be insignificant; but if any almost inconceivably small quantity of that water form a column in a tube so as to communicate its pressure to the thin sheet so spread over the large surface, the pressure at any and every point of that surface may be considerable, and the same as if the whole vessel were filled to the same height as the small tube.

Another form in which the hydrostatic paradox may be stated is, that any quantity of fluid however small may be so employed as to sustain any weight however large (Art. 53), or that any force however small may, by the transmission of pressure in a fluid, be in equilibrium (Art. 27) with or balance any other pressure however

great. Both these propositions are but enunciations of the same truth or law of fluids, which has already been derived from or shewn to be the consequence of the fundamental and characteristic property of the transmission of pressure.

56. *Artesian Wells.* The principles and laws of fluids which have been treated of in the preceding chapters, may be illustrated by, and afford an explanation of, several natural phenomena, particularly those of artesian wells, and natural springs and fountains. In sinking wells, or boring for water, it frequently happens that the water will suddenly break out and spout above the surface of the ground, and continue to play as a natural fountain. The crust of the earth through which the sinking of the well or boring takes place consists of strata of different materials, thickness, and inclinations, some of which are pervious and others impervious to water.



Let the accompanying figure represent the disposition of the strata of which ABC is a bed of sand or other materials readily pervious to and full of water, and DBE a bed or stratum of clay or stone impervious to water, and suppose above this impervious bed various soils more or less pervious to water. Let HB represent a well that is sunk or a boring; then inasmuch as water will rise to its level (Art. 34), and at B there will be a pressure upwards

due to the height of the water in AB, the water will rise in a pipe to the height Hg, or spout out at H, and continue to play as a natural fountain, the height of which will depend on the pressure at H or the level of the water in A. In practice the water will not rise quite to g, the level of the water in the bed AB, owing to the resistance opposed by the sand to the motion of the water. The same pressure which exists at B will exist at other points on the same level, and give rise to natural springs at a, b, c, as shewn by the dotted lines. The height to which the water will spout above B will be a measure of the pressure at that point*.

57. Various other phenomena connected with the supply of water at the surface of the earth, and the flow of rivers, may be fully explained in a similar manner to the preceding. For instance, it not unfrequently happens that the height to which the water rises in natural springs becomes suddenly diminished, or they fall off altogether or vary much at different periods of the year, and in different years; all which fluctuations result from an alteration in the pressure at B, the outlet for the spring from the underground reservoir or river, or sheet of water. This alteration may be occasioned by other wells or borings having been made to tap the same water, or from a diminished fall of rain, the source of all springs, or from some well or boring into the stratum ABC having been sunk through the water-tight stratum into another pervious bed beneath and the water of the bed ABC being drawn off by some easier and more direct course.

Whatever may be the precise circumstances of each particular case, the student must bear in mind this general law as applicable to all supplies of water from underground, that the supply results from the pressure at some point in

* See further relating to springs in the Chapter on Evaporation and Rain.

the interior, due to a head of water retained in a pervious bed of greater or less length and inclination.

The pressure may result from a sheet of water or a column, of small inclination, but many miles in length, so that one extremity of the column is considerably above the other. The resistance of the materials to the passage of the water frequently prevents, as has been already mentioned, the pressure at any point in the interior being that which is due to the whole head, and the same cause frequently occasions the surface of the underground current to have an inclined position instead of being horizontal, the degree of inclination depending on the amount of resistance and the rapidity with which the water is withdrawn at the lower extremity.

The application of the preceding principles will frequently be of service in the investigations of geology, as explaining some of the phenomena which present themselves, and affording evidence as to the extent and inclination of particular strata.

58. *Flow of rivers.* The proposition already established may also be applied to explain the motion of water in rivers and artificial channels. We have seen that the surface of a fluid at rest is a level surface, and a surface of equal pressure, and conversely that when the surface of a fluid is level, the fluid is at rest (Art. 33), also when the pressure is the same at all points of a surface, the fluid is at rest (Art. 31). When therefore a fluid is not at rest the above conditions do not obtain, and the rapidity of the flow of the water depends (other things being the same) on the difference of the pressure at consecutive points. In a river in motion the surface has a certain inclination; hence since each point is pressed by the weight of the superincumbent column, the difference between any two columns is the measure of the force which accelerates the motion of the particles.

The inclination requisite for or corresponding to a given velocity, will depend on the degree of resistance opposed to the motion of the particles; thus when water flows through a bed of sand, as referred to in the preceding article, the surface of the water will have a considerable slope*.

59. *Flow of water in pipes.* The same principles are also applicable to explain the flow of water in pipes; the velocity, other things being the same, will depend on the difference of the levels at the two extremities, as this will be the height of the head available for giving motion to the water. The character of the pipe, as to dimensions, the number of angles, and materials, will affect the result produced by the same head; so that the circumstances of each case must be carefully examined.

* See further on this subject in the Chapter on the Motion of Fluids.

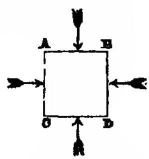
CHAPTER V.

ON THE EQUILIBRIUM OF FLOATING BODIES.

60. In the preceding Chapter the pressure of a fluid on the bottom and sides of the containing vessel, and on a plane immersed in the fluid, was considered. We shall now proceed to treat of the equilibrium of bodies immersed and floating in a fluid, for the purpose of determining the conditions under which they will be at rest and in motion; and if at rest, whether their position is one of stability or instability, and if in motion, the circumstances under which they rise or sink in the fluid.

61. Prop. *A body at rest immersed in a fluid is pressed upwards by a pressure equal to the weight of the fluid displaced.*

(1) Let the body be of a regular figure, as a cube, and wholly immersed in a fluid, and let the upper and lower forces AB and CD be horizontal or parallel to the surface of the fluid, then the sides AC and BD will be perpendicular to that surface. Each of the four sides will be subject to the pressure of the surrounding fluid, and this may be represented by a single force. The pressure on the side AC will be equal to the pressure on the side BD, each being equal to a column of the same base and height. The pressure on the upper and lower faces respectively will be equal to the weight of a column whose base is the area and whose height is the depth of those faces respectively below the surface of the fluid, and the pressure on the lower face



CD will exceed the pressure on the upper face AB by the weight of a column of the fluid whose base is CD and height AC; that is, the weight of the fluid displaced or occupied by the solid immersed. If the weight of the fluid displaced be equal to the weight of the solid immersed, the excess of the pressure on the lower face tending to make the body ascend will be counterbalanced by that weight, and the body will be at rest; but if these weights be unequal, the body will sink or rise according as its weight is greater or less than the excess of the pressure on its lower face.

(2) Let the body be of any shape whatever, and wholly immersed in the fluid. Now since any portion of a fluid may be supposed to be detached from the rest and to become solid, we may suppose the fluid which occupied the space of the immersed body to be so detached and solidified; then this solidified mass will be supported by the surrounding fluid in the same manner as the immersed body is supported, and the upward pressure of the fluid being the same in both cases, the weight supported will be the same, or the body will be pressed upwards by a pressure equal to the weight of the fluid displaced.

62. Prop. *When a body is immersed or floats in a fluid, the horizontal pressures are in equilibrium with each other.*

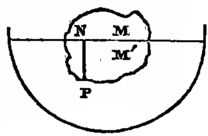
The pressure at all points in the same horizontal plane being equal, and proportional to the depth of the plane below the surface of the fluid (Art. 42), and perpendicular to the surface of the immersed or floating body (Art. 51), if the horizontal forces are not in equilibrium with each other, the body cannot be at rest in the fluid, but will move in a horizontal direction; but the body is at rest, and consequently the pressures in the horizontal direction are in equilibrium with each other*.

* See *Theory of Fluids*, Art. 55.

63. *Plane of floatation.* When a body floats in a fluid, the plane in which the horizontal surface of the fluid intersects the body, is called the *plane of floatation.*

64. Prop. *If a body floats in a fluid, it displaces as much of the fluid as is equal to the weight of the body, and the force with which it presses downwards and is pressed upwards is equal to the weight of the fluid displaced.*

Let the figure represent any body floating in a fluid. Let M be the mass of the body above the surface of the fluid, and M' the mass of the body below the surface of the fluid, then $M+M'$ is the whole mass. Now this body is supported in some way or other by the fluid,



and before the body was immersed, there was a quantity of the fluid equal in bulk to M', which being conceived to be detached from the rest (Art. 30), was supported in the same manner. Each of these then must be supported by the same pressure acting upwards. Since then the weight of the fluid displaced would be sustained by the same upward pressure as sustains the weight of the body, the weight of the displaced fluid and of the body must be equal to each other.

The pressure at any point P is (Art. 43) the weight of a vertical column PN of the fluid, and the whole pressure on the under surface of the floating body is the sum of all such columns. But the sum of these columns makes up the fluid displaced. Hence the downward pressure of the body and the upward pressure of the fluid each equals the weight of the fluid displaced.

Let V be the volume of the floating body, that is, the number of units of bulk it contains, and ρ its density. Then $M+M'=V\rho$, and $V\rho g$ is the weight of the body.

Similarly let V' be the volume of the part immersed, and ρ' the density of the fluid.

Then the weight of the fluid displaced $= V'\rho'g$, and, by the preceding, $V'\rho'g = V\rho g$; $\therefore$ $V'\rho' = V\rho$.

Let $\rho' = \rho$, then $V' = V$, or the whole body is immersed, and will rest in any position, provided it be wholly immersed.

Cor. 1. Since $V'\rho' = V\rho$, we have, $V' : V :: \rho : \rho'$; or the part immersed bears a constant ratio to the whole body.

Cor. 2. Whence it is evident that if a heavy body, as a bucket full of water, be suspended in water by a string, it is pressed upwards by a pressure equal to the weight of the fluid displaced, and therefore water weighs less when surrounded by its own medium than when surrounded by air. Thus it appears that the loss of weight consequent upon the immersion of a heavy body in water is owing to the gravitation of a fluid in *proprio loco* (Art. 9).

65. *Green's Canal Lifts.* A very ingenious and useful practical application of the law that a floating body displaces as much fluid as is equal to its own weight, is afforded by the Canal Lifts* for passing laden boats from one level to another on a canal by raising or lowering them vertically. Two similar cradles or boxes with moveable water-tight doors at each end are suspended by chains over wheels so as to balance each other. These cradles are of a sufficient size and depth to hold, and when filled with water to float, the laden canal-boats. As these cradles balance each other over wheels, a slight excess of water-weight in one of them, or suitable machinery attached to one of the wheels over which they are balanced, will cause one cradle to descend and the other to ascend so as to occupy the lowest and highest positions respectively,

* Designed and erected by Mr James Green, on the Grand Western Canal, near Taunton. See *Trans. Inst. Civ. Engin.*, 4to, Vol. II. p. 185.

and to be on a level with the water in the lower and upper levels of the canals. In this state of things the moveable door at one end of each of the cradles is opened and a free communication made between the water in the cradles and the water in the canal at the two levels, and the cradles may be filled with water from those levels respectively. The cradles being so filled with water a boat may be passed from each of the levels into the cradle with which it communicates, when a quantity of water, equal to the displacement of the boat will flow out of the cradle into the canal, and the moveable doors being then closed the cradles with the boats floating in them may be set in motion by suitable machinery and change their relative situations, the cradle which was at the higher level having descended to the lower, and the cradle which was at the lower having ascended to the higher level. The other moveable door of the cradles being opened the boats so raised from the lower level may be passed out of its cradle into the canal at the upper level, and the boat so lowered from the upper level passed out of its cradle into the canal at the lower level; and other boats being passed in the same operation of transferring a boat from the one level to the other is repeated, the cradles with their boats ascending and descending alternately. If there is no boat ready to be sent up the canal at the time when one is to be sent down, or *vice versâ*, then the one cradle contains its complement of water only, the weight of the cradle being the same whether it contains water only, or water with a boat floating in it, the depth of the water in the cradle being always the same; any excess in the quantity of the water in the cradle being corrected by a portion of the water in the cradle flowing into the canal when the moveable door of the cradle is open, and any deficiency of water in the cradle being

supplied from the canal. By this adjustment of the depth of the water in the boat so as to balance the weights across the wheels, small canal-boats may be passed from one level to another with facility by a lift of from 50 to 100 feet, the loss of water from leakage or waste being small. It will be observed, as a consequence of this ingenious application of the above proposition, that a quantity of water exactly equal in weight to the gross tonnage of the boats and cargo will pass either up or down the canal in a direction contrary to that of the load, i.e. if the trade be all down the canal and the boats return empty, a quantity of water equal to the whole weight of the loading passed down will have passed up from the lowest level of the canal to the highest; and *vice versâ*, if the trade were all upward, the same quantity of water would have passed down the canal from the top to the bottom level independently of waste; and a similar observation applies to the difference or balance of tonnage between the up and down trade either way, since a quantity of water equal in weight to such difference will pass in a contrary direction; and hence arises a somewhat curious proposition, that with such lifts and a downward trade water equal in weight to the loads passed down would be absolutely carried up from the lowest to the highest level of the canal.

66. Prop. *If a body be not at rest in a fluid, the moving force by which it ascends or descends is the difference between the weight of the solid and the weight of an equal bulk of the fluid.*

The condition of equilibrium, namely $V'\rho'g = V\rho g$, is of course not satisfied here, but the body being wholly immersed $V' = V$ and $V\rho'g$ is greater or less than $V\rho g$, according as ρ' is greater or less than ρ.

Let ρ' be greater than ρ, then $V\rho'g$ is greater than $V\rho g$, that is, the weight of the fluid displaced is greater than the weight of the body, or the body is pressed upwards by a

greater force than that with which it presses down. The moving force* upwards is $V\rho'g$, and downwards is $V\rho g$, and the difference of these, or

$$V\rho'g - V\rho g,$$

is the moving force with which it ascends. Similarly, $V\rho g - V\rho'g$ is the moving force with which it descends, if ρ is greater than ρ'. And the accelerating force being the moving force by the mass moved, is

$$\frac{V\rho g \sim V\rho'g}{V\rho} = \frac{\rho g \sim \rho' g}{\rho} = g\left(1 - \frac{\rho'}{\rho}\right).$$

67. *Camel.* On the principle explained in this proposition is founded the contrivance called the *Camel*, for raising sunken vessels, or for lifting vessels over shoals and sand-banks. Two large chests when filled with water are easily sunk, one on each side of the vessel which is to be lifted, and made fast to the side of the keel by straps passing underneath. The water is then pumped out, and the buoyancy of the chests will raise the vessel.

Life preservers also act on the same principle; for a tube impervious to water and inflated with air, is so much lighter than its own bulk of water, that when attached round the waist, it gives the body such buoyancy that the upper parts of the person will be kept above water.

68. *Ascent of light bodies.* The preceding propositions may also be illustrated by the ascent of light bodies, as an air-bubble and a fish in water, or a balloon and smoke in the atmosphere, which, as will hereafter be seen, is a fluid presenting the same conditions for the equilibrium of bodies as already described. Thus, if a small bladder filled with air be inserted in a tall jar containing water or other fluid, and covered with a bladder so that the water in the vessel may be compressed by the pressure of the hand on the

* Whewell's *Elementary Mechanics*, Art. 100. Sixth edition.

covering of the vessel, the air-bubble will sink, and on the pressure being removed rise again. Owing to the diminution in the bulk of the air-bubble by the pressure on the surface of the water, the quantity, and consequently the weight of the water displaced, is diminished, whereas the weight of the bubble remains the same.

The same principle is also illustrated by the rising and sinking of a fish in the water, according as the column of water displaced is increased or diminished on the expansion or contraction of its body.

69. Prop. *If a body float in a fluid, the line joining the centres of gravity of the body and of the fluid displaced is vertical.*

The downward pressure is the weight of the body, and the upward pressure is the weight of the fluid displaced (Art. 64). These two pressures may be considered as two forces applied at the centres of gravity of the body and of the fluid displaced, so that if G and O' (see fig. Art. 73) be these centres, we have two forces acting on the body in the direction of the arrows in the figure. And it is evident that a body acted on by two such forces cannot be at rest unless the forces are opposite as well as equal; that is, unless G and O' are situated in the same vertical line.

Cor. If the forces act as they are represented in the figure, they constitute, since they are parallel and equal, what is technically termed a couple, and a couple admits of no single resultant*.

70. *Conditions of equilibrium.* The conditions that a floating body should remain at rest are,

(1) *That the weight of the body be equal to that of the fluid displaced,* and,

(2) *That the centres of gravity of the body and of the fluid displaced be in the same vertical line.*

* Potter's *Mechanics*, Art. 15.

The two preceding conditions must obtain when a body floats in a fluid and is in equilibrium; they are not however sufficient to determine the nature of that equilibrium; they relate only to the motion of translation upwards or downwards, but have no reference to the motion of rotation. The motions of which a solid body is susceptible may be reduced to two, which are perfectly independent of each other, and since they may coexist in the same body, the state of the body with respect to each must be determined. Suppose a body floating at rest in a fluid to be struck or pushed in an oblique direction, two motions will be communicated to it, one of translation and the other of rotation. Now in consequence of the former motion and of the action of the fluid the body will make oscillations; that is, will move backwards and forwards about its original position of rest, and finally, by the action of the fluid, be reduced to it.

But the motion arising from the latter will differ with the nature of the floating body. In one case the body will make oscillations about its original position, and finally return to it; in another it will remain stationary, after having moved through a certain angle, and in a third it will have no tendency to return to its original position, but will recede farther from it.

These three cases give rise to the three distinctions in the nature of the equilibrium of floating bodies; and a body at rest is said to have an equilibrium of stability, of indifference, and of instability.

71. *Stable Equilibrium.* The equilibrium of a body is *stable* when it floats permanently in any one position, that is, when having been disturbed and made to revolve through any angle, it has a tendency to resume its former position.

Unstable Equilibrium. The equilibrium of a body is *unstable* when after having been disturbed it has no tendency to return to its former position of equilibrium, but recedes farther from it.

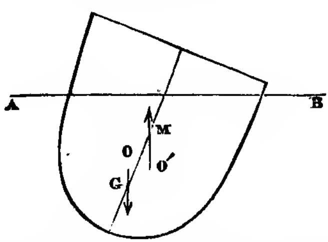
Neutral or Indifferent Equilibrium is when the body rests in every position, and has no tendency either to return to or to recede farther from its original position.

An ellipse placed on the extremity of its axis minor on a horizontal plane returns to its original position if disturbed, but if it be placed on the extremity of its axis major it recedes farther from its original position. In the former case the equilibrium is stable, in the latter unstable. The nature of the equilibrium is a most important inquiry, and the theoretical investigations lead to results of the greatest importance in the building, lading, and ballasting of vessels.

72. *Metacentre.* The point of intersection of the vertical through the centre of gravity of the fluid displaced (the body having been disturbed through a very small angle) with the vertical through the centre of gravity of the body when at rest, is called the metacentre of the body.

73. Prop. *To determine when the equilibrium is stable, unstable, or indifferent.*

Let G be the centre of gravity of a body floating in a fluid whose surface is AB, and O the centre of gravity of



the fluid displaced. Then, when the body floated at rest,

GO was vertical (Art. 69); let the body be very slightly disturbed, so that GO is inclined at a very small angle to the vertical, and let O' be the centre of gravity of the fluid displaced. Let the vertical through O' meet OG in M, then M is the metacent of the body. Let us consider the forces acting on the body in this position. There is the weight of the body acting downwards through G, and of the fluid acting upwards through O' in the direction $O'M$, both of which tend to bring the body back to its original position. Hence in this case the equilibrium is stable, and the metacentre is above the centre of gravity of the body. Hence, when the metacentre is above the centre of gravity, which is the case when the body is heavily loaded at the lowest parts, the pressure of the fluid upwards, and the weight of the body downwards, both tend to bring the body back into its original position, and the equilibrium is therefore stable.

But if the metacentre be below the centre of gravity of the body, the pressure of the fluid upwards and the weight of the body downwards both tend to move the body farther from its original position, and the equilibrium is therefore unstable.

Lastly, if the metacentre and centre of gravity coincide, the forces being equal and applied at the same point in opposite directions, the body will rest in that position, and the equilibrium is indifferent.

Hence the equilibrium of a floating body is stable, unstable, or indifferent, according as the metacentre is above, below, or coincident with the centre of gravity of the body.

Cor. Hence the necessity of having not only the heavier parts of a ship's cargo stowed at the bottom of the vessel, but also of having the vessel ballasted or the keel heavily laden, is apparent. For the masts and rigging may raise the centre of gravity of the vessel above the metacentre, in which case the vessel is in a position of unstable

equilibrium, and will be brought on her beam ends by the slightest breeze. The danger from the rolling of large vessels arises from the liability of parts of the cargo to shift, in which case the equilibrium may cease to be stable; and the danger of standing up in a small boat is quite apparent, should the elevation of the body raise the centre of gravity of the floating mass above the centre of gravity of the fluid displaced.

73 *a.* PROP. *The positions of stable and of unstable equilibrium recur alternately.*

This is a general proposition applicable to all cases of equilibrium, and not confined to bodies floating in a fluid. Let the body be made to revolve out of one position of stable equilibrium into another. Now immediately adjacent to its first position, its tendency is to return to it, or to check the revolution; and immediately adjacent to its second position, its tendency is from the first position, or to continue its revolution. There is therefore some intermediate position, in which this tendency changes its direction. Suppose the body to be in this position; then whichever way it is disturbed it moves from this position, or this position is one of unstable equilibrium.

CHAPTER VI.

ON SPECIFIC GRAVITY.

74. One substance is said to be specifically heavier or lighter than another, when being equal as to magnitude the weight of one exceeds or falls short of the weight of the other. Thus iron and mercury are specifically heavier than water, for the weight of a cubic inch of iron is greater than the weight of a cubic inch of water, and the weight of a pint of mercury is greater than the weight of a pint of water. Thus also cork and wood are specifically lighter than iron. And the fact that the weight of any particular substance is greater or less than the weight of the same bulk of some other substance, is expressed by saying, that the specific gravity of the one substance is greater or less than the specific gravity of the other.

75. *Definition of Specific Gravity.* The specific gravity of a substance is the ratio of the weights of equal bulks of that substance, and of some other substance with which it is compared.

But insuperable difficulties present themselves in comparing some substances with each other, and this comparison can in many cases only be effected by taking some standard substance which is subject to little variation, and with which every other substance may be readily compared; so that the substances, being compared immediately with this, may be compared mediately with each other. Now water possesses these qualities in a remarkable degree; and, in order to remove all possible sources of inaccuracy, distilled water, at a temperature of 60^0, is used as the standard medium.

In comparing the specific gravity of different gases, it would be inconvenient, in consequence of their extreme lightness, to compare them only with water; hence pure and dry atmospheric air, at a temperature of 60^0, and the barometer at 30 inches, is used as the medium of comparison.

Hence it appears that the specific gravity of a solid or liquid is found by dividing the weight of a portion of the substance by the weight of an equal bulk of water, and the quotient obtained by performing the division to a sufficient number of decimal places is set down as the specific gravity of the substance. Similarly the specific gravity of a gas is expressed by the quotient obtained by dividing the weight of a portion of the gas by the weight of an equal volume of atmospheric air. And a *table of specific gravities* consists of these quotients so set down for all known substances.

76. *Tables of Specific Gravities.* It is particularly to be observed, that these quotients, obtained by dividing the weights of equal volumes of two substances, and set down in the Tables, express the *number* of units of weight in a certain volume of the substance. And the unit of weight which is here employed is the weight of that volume or bulk of water which is the unit of volume of the substance, whatever that unit may be. If then the substance be measured in cubic inches, the unit of weight is the weight of a cubic inch of water, and the specific gravity of any substance is the number of cubical inches of water which is equal in weight to one cubical inch of the substance. And the quotient which expresses the specific gravity of any substance means the volume of water which is equal in weight to that unit of volume by which the substance is measured.

The specific gravity of mercury as set down in the Table* is 13·58; that is, 13·58 cubic inches or cubic feet of water weigh

* See Table of Specific Gravities at the end of the Volume.

as much as one cubic inch or cubic foot of mercury. Hence also it is evident that mercury is 13·58 times heavier than water. Thus the *absolute weight* of substances may be determined from a table of Specific Gravities, if the unit of weight, that is, if the weight of a quantity of the water equal to the unit of bulk, be known.

77. *The specific gravity of different substances is as the density of the substances.*

Let V and V' be the volumes of two bodies A and B; ρ and ρ' their densities, then the weights of the bodies are $V\rho g$ and $V'\rho' g$ (Art. 13).

Let the density of the standard fluid be unity, then the weights of portions of it equal in bulk to V and V' are Vg and $V'g$.

$$\text{Hence the specific gravity of } A = \frac{V\rho g}{Vg},$$

$$\text{and of } B = \frac{V'\rho' g}{V'g},$$

$$\text{or sp. gr. of } A : \text{sp. gr. of } B :: \frac{V\rho g}{Vg} : \frac{V'\rho' g}{V'g} :: \rho : \rho',$$

or as the density of the substances.

Cor. Hence the specific gravity of different substances is as the weights of equal bulks of the substances.

78. Prop. *When a solid is immersed in a fluid, the weight lost is to the whole weight of the body as the specific gravity of the fluid is to that of the solid.*

When a body is immersed in a fluid, the moving force with which it descends, or tends to descend if sustained, is equal to the difference between the weight of the solid and the weight lost by the action of the fluid. Let W be the weight of the solid and W' the weight lost by the action of the fluid; the moving force then with which it descends is $W - W'$.

Now this force is the difference between the weight of the solid and the weight of the fluid displaced (Art. 64), and therefore if w be the weight of the fluid displaced, the force is $W-w$;

$$\therefore\ W-W'=W-w;\quad \text{or } W'=w,$$

or the weight lost by the action of the fluid is equal to the weight of the fluid displaced; that is, it is equal to the weight of a quantity of the fluid equal in bulk to the body immersed.

Let V be the volume of the body immersed, ρ the density of the body, and ρ' of the fluid.

Then $W=V\rho g$ and $w=V\rho' g=W'$...... (Art. 13);

$$\therefore\ W' : W :: V\rho' g : V\rho g$$
$$:: \rho' : \rho.$$

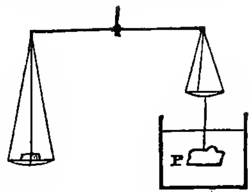
But the specific gravity of bodies is as their density (Art. 77); therefore the weight lost : the weight of the body :: the specific gravity of the fluid : the specific gravity of the solid.

Also when the body is not wholly immersed, we have the volumes of the part immersed and of the whole body in the same ratio (Cor. 1, Art. 64).

If the body be lighter than water, a known weight must be employed to sink it, and the same proposition will be found to hold. Many applications of this proposition occur in the practical determination of the specific gravity of bodies.

79. *Hydrostatic Balance.* The specific gravity of a solid body is very readily found by the hydrostatic balance, which in its simplest form is a common pair of scales, with a fine thread attached to the under surface of one of the scale pans. The substance whose specific gravity is required is weighed in air, and then, being attached to the thread, is immersed in distilled water at the standard temperature, and again weighed.

Let w be the weight of the substance P in air, and w' its weight when immersed in the water.



Then $w - w'$ is the weight lost, which is equal to the weight of the fluid displaced, and the fluid displaced is equal to the bulk of the substance immersed. Hence $w - w'$ is the weight of the fluid equal in bulk to the substance whose weight is w. The specific gravity therefore of the substance $= \dfrac{w}{w - w'}$, (Art. 75).

But if P be lighter than water, as if it be a piece of dry wood, it will not sink. That it may sink in water, let a heavy substance P_1 be attached to it, which weighs w_1 in air and w_2 in the water.

Let w' be the weight of $P + P_1$ in the water.

Then weight of water equal in bulk to $P + P_1 = w + w_1 - w'$;

and .... $P_1 = w_1 - w_2$;

$\therefore$ .... $P = w + w_2 - w'$;

$$\therefore \text{ the specific gravity of } P = \frac{w}{w + w_2 - w'}.$$

If the substance be soluble in water, it must be enclosed in a lump of wax of known weight and magnitude, and its specific gravity may be found in a manner similar to the last case.

80. Prop. *To find the specific gravity of a compound.*

Substances of the same kind have invariably the same specific gravity, and this furnishes a test whereby we may determine whether two similar substances are the same or different. But the specific gravity of a compound will in general be different from the specific gravity of either of the component substances, and may be found when the whole quantity of matter in the compound is the sum of the matter in the parts.

Let V, V' be the volumes of the component substances, ρ and ρ' their densities, and ρ'' the density of the compound, then

$$(V+V')\,\rho''=V\rho+V'\rho',$$

$$\text{or } (V+V')\,\frac{\rho''}{1}=V\frac{\rho}{1}+V'\frac{\rho'}{1};$$

but specific gravity is as the density (Art. 77). Hence if the density of the water be unity, and s, s', s'', be the specific gravities of the component substances and of the whole substance, then

$$(V+V')\,s''=Vs+V's';$$

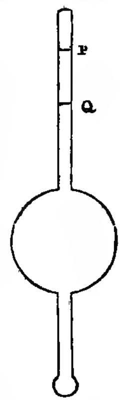
$$\therefore s''=\frac{Vs+V's'}{V+V'}.$$

Thus in making an alloy of metals, the specific gravity of the compound is known from the specific gravity of the component metals, (if the density of the metals undergoes no change by the admixture); and if the alloy be improperly made, the adulteration may be detected.

It is said that Archimedes discovered that the gold which had been given by Hiero to an artificer to make into a crown, was not put pure into the crown, but alloyed with a baser metal. The adulteration must have been detected by the principles explained in the preceding articles.

This proposition is not true in cases of chemical combinations, for it frequently happens that the volume of a chemical combination is less than that of the two combined substances. Thus, if a pint of water and a pint of sulphuric acid be mixed together, the mixture will not make a quart.

81. *Common Hydrometer.* The simplest instrument for finding the specific gravity of a fluid is the common hydrometer. A hollow sphere is pierced by a stem, the upper portion of which is graduated, and the lower portion loaded so that the instrument may float vertically when placed in a fluid. It will also have great stability, for the centre of gravity will be below the metacentre (Art. 73).



Let V be the whole volume of the instrument, and k the section of the graduated stem.

When placed in one fluid (A) let it sink to P, and in another fluid (B) let it sink only to Q. Let the zero point of the graduation be at the top of the stem, and let P be distant x divisions from the top of the stem, and Q be distant y divisions from the same point.

Let ρ and ρ' be the densities of the two fluids.

Then the quantity of the fluid (A) displaced, is $(V-xk)$,
and .... (B) .... $(V-yk)$,

and the weights are respectively

$$g\rho(V-xk), \text{ and } g\rho'(V-yk).$$

Now the weight of the fluid displaced is equal to the weight of the body immersed (Art. 64), hence since the same instrument is used for both fluids,

$$g\rho(V-xk)=g\rho'(V-yk),$$

or $V-xk : V-yk :: \rho' : \rho$

:: the sp. gr. of (B) : the sp. gr. of (A)
(Art. 77);

therefore the specific gravity of (A)

$$=\frac{V-yk}{V-xk}\cdot \text{sp. gr. of } (B).$$

Let the fluid (B) be the standard fluid, then its specific gravity will always be represented by unity, and therefore if the instrument sinks to P in any other fluid, the specific gravity of that fluid is $\frac{V-yk}{V-xk}$, and the value of this ratio for every value of x being registered in tables, the specific gravity required is known by the inspection of these tables.

82. *Sikes's Hydrometer.* The value of some wines and of all spirituous liquors, depends principally on the relative quantity of alcohol and water which they contain. Hence it is a matter of the greatest importance to merchants and revenue officers to be able to determine with facility this proportion. This is done by the hydrometer, which shews at once whether spirits are above or below proof. For proof spirit consists of equal portions of pure spirit or alcohol, and of water. Now water is heavier than alcohol; if therefore there be more water than alcohol, the hydrometer will not sink so deep as in proof spirit; if there be less, the hydrometer will sink deeper; in the one case the mixture is below proof, and in the other it is above proof, and the degree of adulteration may evidently be known from a table calculated in the manner described in the last article. The instrument generally used for this purpose is the hydrometer invented by Sikes.

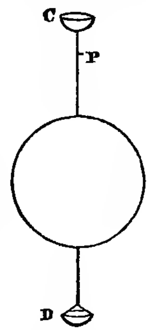
The only difference between this instrument and the common hydrometer, consists in the stem, which is thin and flat. It is accompanied with eight small weights, which may be placed on the stem so as to increase the weight of the instrument, if requisite, when the specific gravity of a heavy fluid is required. For the instrument being used for determining the specific gravity of very light fluids would not sink to the lowest division on the graduated stem in a heavy fluid, without the addition of

one of these weights. The table used is different for each different weight placed on the instrument.

83. *Nicholson's Hydrometer.* This hydrometer serves to determine the specific gravity both of solids and of fluids. A hollow sphere is pierced by a stem graduated as usual, and carrying a cup at each end, the lower one of which is loaded so as to insure the stability of the instrument in a vertical position.

This instrument is always sunk to the same depth by weights placed in the upper cup.

(1) *Specific gravity of a solid.* Let W be the weight of the instrument, and W' the weight of the fluid displaced when the instrument is sunk to the standard depth, and w the weight placed in the upper cup to sink it to that depth.



Let X be the weight of the body whose specific gravity is required, and Y the weight of a bulk of the fluid equal to the bulk of the body. Then,

$$\text{specific gravity of the body} = \frac{X}{Y}.$$

The body being first put in the upper cup, let the instrument be sunk to the given point P by a weight w' in the upper cup. Then we have the following equations:

$$W' = W + w \ldots\ldots\ldots\ldots\ldots\ldots (1),$$
$$W' = W + X + w' \ldots\ldots\ldots\ldots (2);$$

whence, by subtraction,

$$X = w - w'.$$

Next, let the body be placed in the lower cup, and now let a weight w'' in the upper cup sink the instrument to the given depth. Then,

$$W' + Y = W + X + w'' \ldots\ldots (3).$$

Subtracting equation (2) from this, we have

$$Y = w'' - w';$$

$$\therefore \text{ specific gravity} = \frac{X}{Y} = \frac{w - w'}{w'' - w'}.$$

And the specific gravity of a body being the ratio of the weights of equal bulks of the body and of the fluid, the specific gravity required $= \dfrac{w - w'}{w' - w_{\prime}}$.

(2) *To compare the specific gravity of two fluids* A *and* B.

Let W be the weight of the instrument, and w the weight requisite to sink it to the point P in the fluid (A), and w' in the fluid (B).

Then the weight of the fluid (A) displaced $= W + w$,
and .... (B) .... $= W + w'$,
and the quantity is the same in each case, since the instrument is sunk to the same point. Now the specific gravities of two fluids are in the ratio of the weights of equal bulks of the fluids (Cor. Art. 77).

$\therefore$ the sp. gr. of A : the sp. gr. of B :: $W + w : W + w'$.

Let the fluid (B) be pure water at the standard temperature, the specific gravity of the fluid $(A) = \dfrac{W + w}{W + w'}$.

84. In the preceding propositions the weight in air has been taken as the true weight. This is not quite exact, for the air being a fluid, the weight in air is only the apparent weight, the true weight being the weight *in vacuo;* so that the preceding propositions are accurately true only on the hypothesis of the bodies having been weighed *in vacuo.* The corrections are, however, very small, and the difference in the specific gravity of bodies from this circumstance can only be detected by very nice experiments. The corrections requisite to render the

preceding proposition accurate are given in the following articles.

Also, in very nice experiments, great care must be taken to observe the temperatures at which the observations are made, since every body expands by heat and contracts by cold, and will therefore occupy more space, that is, be of larger volume at one time than at another.

The height of the barometer must also be noted, and the results of different observations must all be reduced to the same standard.

85. PROP. *To find the specific gravity of air, that is, to compare the density of air and water.*

Let a large flask or bottle full of the air whose specific gravity is required be weighed, and let w be its weight, and let w' be its weight when full of water, and $w_,$ its weight when the air is exhausted.

Then $w - w_,$ is the weight of the air which the flask holds,
and $w' - w_,$ .... water ....

$$\therefore \text{ the sp. gr. of the air} = \frac{w - w_,}{w' - w_,}.$$

COR. The specific gravity of the gases will be found in the same manner by weighing a large flask when full and empty, and thus finding the weights of equal bulks of gas and air.

86. PROP. *To determine the true weight and specific gravity of a substance.*

Let w be the weight of the body when weighed in air, and w' its weight when weighed in water.

Let m be the specific gravity of air, and s the apparent specific gravity of the substance, as determined by one of the methods explained in the preceding propositions.

Let x be the true weight of the body, and y its true specific gravity (Art. 84).

The weight in air (w) = the true weight (x) − the weight of an equal bulk of air;

and the weight of an equal bulk of air : the true weight of the body (x) :: specific gravity of air (m) : the true specific gravity of the body (y);

$$\therefore\ w = x - \frac{mx}{y} = x\left(1 - \frac{m}{y}\right) \dots\dots\dots\dots\dots (1).$$

The weight in water (w') = the true weight (x) − the weight of an equal bulk of water;

and the weight of an equal bulk of water : x :: 1 : y as before;

$$\therefore\ w' = x\left(1 - \frac{1}{y}\right) \dots\dots\dots\dots\dots (2).$$

$$\text{Then from (1) } \frac{m}{y} = 1 - \frac{w}{x},$$

$$\text{and } \dots\dots (2)\ \frac{1}{y} = 1 - \frac{w'}{x};$$

$$\therefore\ m = \frac{1 - \dfrac{w}{x}}{1 - \dfrac{w'}{x}},$$

$$m - \frac{mw'}{x} = 1 - \frac{w}{x};$$

$$\therefore\ \frac{w - mw'}{x} = 1 - m; \quad \therefore\ x = \frac{w - mw'}{1 - m}.$$

$$\text{or } x = (w - mw')(1 + m) \text{ nearly},$$

$$= w + m(w - w') \text{ nearly},$$

since all powers of m above the first may be omitted. Hence the true weight is greater than the apparent by $m(w - w')$.

$$\text{Again, dividing (1) by (2), } \frac{w}{w'} = \frac{y - m}{y - 1};$$

$$\therefore\ y = \frac{w - mw'}{w - w'} = \frac{w}{w - w'}(s) - \frac{mw'}{w - w'}.$$

Hence $\frac{mw'}{w-w'}$, must be subtracted from the apparent specific gravity, and the result is the true specific gravity of the substance.

87. *On the construction and use of a table of specific gravities.*

In the preceding articles several instruments for determining the specific gravities of substances have been explained. A few examples in which the data obtained by some of these methods are made use of, may serve to render the subject more intelligible, and to explain fully the method of constructing a table of specific gravities, and its use when constructed.

Ex. 1. A piece of copper weighs 31 grains in air and $27\frac{1}{2}$ grains in water, as ascertained by the hydrostatic balance (Art. 79).

Then in the case $w = 31$, and $w' = 27\frac{1}{2}$;

$$\therefore \text{ the sp. gr. of copper} = \frac{31}{31 - 27\frac{1}{2}} = \frac{62}{7} = 8{\cdot}85,$$

which coincides very nearly with the value given in the table of specific gravities.

Ex. 2. A piece of dry elm weighs 920 grains, let it be varnished to prevent its absorbing any water by which its bulk would be increased, and when attached to a piece of metal which weighs 911 grains in water, let it weigh in water 331 grains.

Then, by Art. 79,

$$s = \frac{w}{w + w_2 - w'} = \frac{920}{920 + 911 - 331} = \frac{920}{1500} = {\cdot}613,$$

the specific gravity required.

Ex. 3. Two metals, whose specific gravity is known, are mixed in a known proportion; to find the specific gravity of the compound.

By (Art. 80) $s'' = \dfrac{Vs + V's'}{V + V'}$.

Let V be twelve measures of gold and V' one of copper, then, referring to the table, $s = 19{\cdot}25$, and $s' = 8{\cdot}9$;

$$\therefore\ s'' = \frac{12 \times 19{\cdot}25 + 8{\cdot}9}{12 + 1} = \frac{239{\cdot}9}{13} = 18{\cdot}45.$$

This is very nearly the specific gravity of the sovereign, and would be accurately so if gold and copper did not change their densities in the least degree on forming a compound.

Ex. 4. A Nicholson's hydrometer, weighing 250 grains, requires 72 grains to sink it to the given depth in water, and 9 grains in alcohol.

Then (Art. 83) the specific gravity of alcohol $= \dfrac{W + w}{W + w'}$

$$= \frac{250 + 9}{250 + 72} = \frac{259}{322} = {\cdot}804$$

which agrees very nearly with the table.

Ex. 5. When the thermometer is at 60^0, and the barometer at 30 inches, 100 cubic inches of dry atmospheric air weigh 31·0117 grains. And 100 cubic inches of oxygen weigh 34·109 grains;

$$\therefore \text{ (Cor. Art. 77) the sp. gr. of oxygen} = \frac{34{\cdot}109}{31{\cdot}0117}$$

$$= 1{\cdot}1025,$$

and the specific gravities of the other gases are found in the same way.

Ex. 6. To find the weight of a cubic inch of mercury. The weight of a cubic inch of mercury equals the weight of 13·58 cubic inches of water (Art. 76). Now a cubic foot of pure water, at a temperature 60^0, weighs very accurately 1000 ounces avoirdupois.

And there are 12^3 or 1728 cubic inches in a cubic foot, therefore a cubic inch weighs $\frac{1000}{1728}$ ounces*. And a cubic inch of mercury weighs $\frac{13{\cdot}58 \times 1000}{1728} = \frac{13580}{1728} = 7{\cdot}85$ ounces. And the weight of any other substance may be found in the same way.

* The pound avoirdupois contains 7000 grains, therefore an ounce

$$= \frac{7000}{16} = 437{\cdot}5 \text{ grains.}$$

A cubic inch of water weighs $\frac{1000 \times 437{\cdot}5}{1728}$ grains

$$= \frac{437500}{1728} = 253{\cdot}17 \text{ grains.}$$

CHAPTER VII.

ON THE LAWS OF ELASTIC FLUIDS.

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88. The propositions in the preceding Chapters have applied equally to all fluids, both liquids and gases, since the properties on which they depended are common to all. But the elastic fluids or gases have some properties which are the necessary consequences of the repulsive forces which exist between their particles, and which have now to be considered. Common air is found to be a perfectly elastic fluid (Art. 16), and any proposition proved for air in consequence of this property will be true for all other elastic fluids.

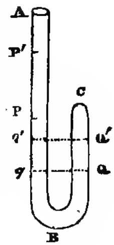
The characteristic of elastic fluids distinguished from liquids, is the pressure which they exert against the sides of any containing vessel, and which is called the *elastic force*. The pressure arising from this elastic force is the same at every part of the fluid in equilibrium, and must be carefully distinguished from the pressure due to gravity, which varies with the position of the point pressed.

89. Prop. *The elastic force of air at a given temperature varies inversely as the space it occupies.*

There are many different ways of establishing this experimental truth, which is called Boyle's or Mariotte's law*; the following admits of great accuracy.

* This law was established by Boyle in 1662, and subsequently by Mariotte. For the deviations from this law in some of the gases which may be readily liquefied, see Graham's *Chemistry*, Second edition, pp. 13 and 76.

Let a glass tube, having one end open and the other closed, be bent into the form ABC, AB being parallel to BC. Let the communication between the two tubes be cut off by a small quantity of mercury placed at B. Then air contained in BC will exert the same elastic force on all equal portions of surface. Let the section of the tube be for simplicity taken as the unit of area. Let p represent the elastic force of the air in BC; this will also be the pressure or weight of the superincumbent column of the atmosphere on the surface of the mercury at B. Let mercury be poured down the leg AB, it will rise in BC, and the air in BC will be compressed, and the mercury in the leg AB will stand higher than the mercury in BC. Let P and Q be relative positions of the mercury in the two legs. Then drawing the horizontal line Qq, the portion Pq of the mercury is balanced by the increased elastic force of the air in CQ. Let ϵ be the elastic force of the air in CQ; and ρ the density of the mercury, then



$$\epsilon = p + g\rho Pq.$$

Let more mercury be poured in, and let P', Q' be the relative positions of the surface of the mercury, then drawing $Q'q'$ horizontally, if ϵ' be the elastic force in CQ',

$$\epsilon' = p + g\rho P'q';$$

$$\therefore \frac{\epsilon}{\epsilon'} = \frac{p + g\rho Pq}{p + g\rho P'q'}.$$

Let the spaces CQ, CQ' be accurately measured, then it will be found that

$$\frac{CQ'}{CQ} = \frac{p + g\rho Pq}{p + g\rho P'q'}.$$

And consequently,

$$\frac{\epsilon}{\epsilon'} = \frac{CQ'}{CQ};$$

that is,

$$\epsilon : \epsilon' :: \frac{1}{CQ} : \frac{1}{CQ'};$$

or the elastic force of the air varies inversely as the space occupied.

In the preceding case we have supposed a quantity of common air detached or cut off from the external air by the mercury at *B*. In establishing the same law for the gases, or more generally for all elastic fluids, it will frequently happen, that the elastic force of the fluid on which we are experimenting is less than that of the external air, and consequently, the mercury will stand higher in the longer than in the shorter leg. Suppose, for instance, in the preceding figure that the tube is closed at *A* and open at *C*, and that the elastic fluid is included in the spaces *AP* and *AP′*, the surfaces of the mercury in the open leg being at *Q* and *Q′*. Then we should have

$$\frac{\epsilon}{\epsilon'} = \frac{p - g\rho Pq}{p' - g\rho P'q'},$$

and the elastic force would as before be found to be inversely as the spaces.

Similarly, when the elastic force of the fluid is greater than that of the external air the same law will be found to be true.

The truth of this result will depend on the temperature being the same throughout the experiment. If the mercury be suddenly poured in the heat developed will produce an elevation in temperature which will at first cause a considerable deviation from the above law.

90. PROP. *The elastic force of any gas at a given temperature is proportional to its density.*

The air being a perfectly elastic fluid, the diminution in volume is proportional to the compressing force (Art. 16).

Now the density increases as the volume decreases (Art. 12), and, by the equality of action and re-action, the elastic force is equal to the compressing force. Therefore the elastic force is proportional to the density.

Let k be the elastic force of air whose density is unity, and p the elastic force, or pressure exerted on a unit of surface by air whose density is ρ. Then

$$p : k :: \rho : 1; \quad \therefore p = k\rho.$$

91. Prop. *To determine the volume of any given quantity of gas at a given temperature.*

From the experiments of Dalton and Gay-Lussac, it appears that all gases under the same pressure undergo equal expansions for equal increments of temperature.

Hence the coefficient or law of expansion (Art. 91), is the same for all gases, and being determined for one is known for all. Now 100 measures of air expand to 137·5 measures on being heated from 32^0 to 212^0 of Fahrenheit's thermometer, that is, 37·5 is the increment of bulk of 100 measures of any gas for an increase of 180 degrees of temperature;

therefore the increment of bulk of one measure for 180^0

$$= \frac{37{\cdot}5}{100} = {\cdot}375,$$

and the increment of bulk of one measure for 1^0

$$= \frac{{\cdot}375}{180} = \frac{1}{480};$$

$$\therefore a = \frac{1}{480} \ \ldots\ldots\ldots \ (\text{Art. } 91).$$

Let V be the volume of any gas at a given temperature τ, and V' its volume at any temperature τ'.

Let V_0 be its volume at 32^0 F.

Then $V = V_0 + a(\tau - 32) V_0 = \{1 + a(\tau - 32)\} V_0;$

$$V' = V_0 + a(\tau' - 32) V_0 = \{1 + a(\tau' - 32)\} V_0;$$

$$\therefore\ V' = \frac{1+\alpha(\tau'-32)}{1+\alpha(\tau-32)}\,V$$
$$= \{1+\alpha(\tau'-\tau)\}\,V$$

very nearly, since α is so small that the terms involving higher powers than the first may in general be omitted*.

92. PROP. *To express the relation betwixt the elastic force, the density, and the temperature, of any gas.*

It is found by experiment that the air and all other gases when subject to the same and a constant pressure dilate equally for equal increments of temperature; namely, by $\frac{1}{480}$th of their volume for each degree of Fahrenheit, (Art. 91).

If then the volume of any gas be constant, its elastic force will increase, and if the elastic force be constant, that is, if it be subject to the same pressure, its volume will increase for every increase of temperature. It is therefore of the greatest importance to connect these quantities by an equation. This law is from the name of its discoverer called Amonton's Law.

Let $V_{,}$ be the given volume of a gas at the standard temperature, $p_{,}$ its elastic force, and $\rho_{,}$ its density.

The elastic force $p_{,}$, that is, the pressure on a unit of surface remaining the same, let the temperature be increased by θ°, let V' be the volume, and ρ' the density of the gas, then if α be the increment of bulk for each degree of temperature,

$$V' = V_{,}(1+\alpha\theta).$$

But the density varies inversely as the volume;

$$\frac{\rho'}{\rho_{,}} = \frac{V_{,}}{V'} = \frac{1}{1+\alpha\theta}; \quad \therefore\ \rho' = \frac{\rho_{,}}{1+\alpha\theta} \ \ldots\ldots\ldots\ldots(1).$$

* See Graham's *Chemistry*, Second edition, p. 13, where it appears that $\alpha = \frac{1}{491}$, according to the most recent and exact experiments.

Now suppose the pressure to be changed, the temperature remaining constant, namely, let p be the value of $p_{\prime}$, and ρ the value of ρ', then by Boyle's law (Art. 89)

$$\frac{p}{p_{\prime}} = \frac{\rho}{\rho'}; \quad \therefore p = \rho\frac{p_{\prime}}{\rho'} = \rho\frac{p_{\prime}}{\rho_{\prime}}(1 + \alpha\theta), \text{ by (1)}.$$

But (Art. 90) $p_{\prime} = k\rho_{\prime}$ or $\frac{p_{\prime}}{\rho} = k$ a constant quantity expressing the ratio between the elastic force and the density at a given temperature, therefore

$$p = k\rho(1 + \alpha\theta).$$

This formula is applicable to all gases, vapours, or their mixtures.

93. PROP. *If any number of different gases be in free communication with each other, or be separated by a porous substance, the gases will diffuse themselves through each other.*

Let any two vessels be taken, one containing carbonic acid gas and the other hydrogen, and let the one containing hydrogen be set above the other, and communicate by an aperture with the lower. Then if the gases in the two vessels be examined, after a short interval, carbonic acid gas will be found in the upper and hydrogen in the lower vessel. If the upper vessel be filled with oxygen, nitrogen, or any other gas, the same phenomena will ensue; the gases will be found, after a short interval, to be in a state of mixture, and will at last be distributed equally through both vessels; and the same results are observed, whatever be the number of vessels containing different gases which are in communication with each other; the gases are all diffused equally through each other, so that any part of any one vessel will contain a portion of all the gases. Again, when gas is contained in a jar which has a crack in it, or in a porous vessel, the gas gradually diffuses itself

into the air, and the air into the gas, each passing through the cracks or pores at the same time, but in opposite directions.

A great number of experiments on the diffusion of gases through the pores of a plug of plaster of Paris have been made by Prof. Graham*, and lead to the conclusion that the diffusion or spontaneous intermixture of two gases in contact is effected by an interchange in position of indefinitely small volumes of the gases, which volumes are not necessarily of equal magnitude, but are in the case of each gas inversely proportional to the square root of the density of the gas.

94. Prop. *The particles of one gas, though highly repulsive to each other, exert no sensible action of repulsion on the particles of another gas.*

This is Dalton's law, and as it serves to explain all the cases of the admixture of gases which present themselves, is far more satisfactory than the theories of chemical combination†. We saw, in the preceding article, that carbonic acid gas ascended through hydrogen, and that hydrogen descended through carbonic acid gas.

Now the specific gravity of carbonic acid is 1·527, and the specific gravity of hydrogen is 0·069. Hence carbonic acid gas is 22 times heavier than hydrogen, and yet they permeate each other, and their particles remain intermingled, which is contrary to all known laws of the mutual action of particles on each other.

Again, in the constitution of the atmosphere, there is a constant mixture of at least five different gases, of very different specific gravities‡, and no apparent tendency to

* *London and Edinburgh Phil. Magazine*, March, April, May 1833; and Graham's *Chemistry*, Second edition, pp. 87, 88.

† Turner's *Chemistry*, p. 270, &c.

‡ Dalton, *Memoirs of the Manchester Society*, Vol. v., and *New System of Chemical Philosophy*, Chap. II. Sect. 2.

separation. Add to which, the refractive power of the atmosphere is precisely such as a mechanical admixture of the gases ought to possess, and different from what would be expected were its elements chemically united. But if Dalton's law be true, and there is no sensible mutual action betwixt the particles of different gases, the compound may exist without any regard whatever to the specific gravity of the component gases. One gas also is as a vacuum with respect to another, and the only opposition which the particles of one gas experience from the particles of another, is a mechanical impediment arising from the inertia of the particles, and subject to the laws of inelastic bodies. Thus one gas permeates and moves through the interstices of the other, as a spring of water flows through a sand bed; and, though during the diffusion the motion is retarded by the inertia of the particles, yet when the mixture is complete, or the gases are in a state of rest, the particles press only on those of their own kind.

95. Prop. *To determine the elastic force of any mixture of gases, supposing that the particles of the gases act only on those of their own kind.*

The principles stated in the preceding article being assumed to be true so far as they refer to the fact of the diffusion of one gas through the other, and that one may consequently be considered as a vacuum with respect to the other, we may find the elastic force of the mixture of any number of gases of different elasticities contained in the same vessel.

Each gas will when expanded have the elastic force due to its volume, and the whole pressure on any part of the containing vessel will be the sum of the elastic forces of the contained gases.

Let V and V' be the volumes of two gases A and B, which communicating with each other become mixed together by diffusion (Art. 94). Let p and p' be the elastic forces which the gases A and B exert on a unit of surface of the containing vessels. Then when the gases are diffused through each other, the gas A occupies a space $V + V'$. Hence, since the elastic force of any gas is inversely as the space it occupies,

the elastic force of A after the diffusion $: p :: \dfrac{1}{V+V'} : \dfrac{1}{V}$;

$$\therefore \text{ the elastic force of } A = p \cdot \frac{V}{V+V'}.$$

Similarly the elastic force of $B = p' \cdot \dfrac{V'}{V+V'}$, and the whole elastic force on any portion of the containing surface is the sum of the elastic forces of the two fluids diffused through the whole space, and acting repulsively on the particles of their own kind.

Let $p = p'$, then,

$$\text{the elastic force of the mixture} = \frac{pV + pV'}{V+V'}$$
$$= p,$$

or the sides of the containing vessel sustain the same pressure from the elastic force of the mixture, as each separate vessel sustained before from the elastic force of the gas which it contained. This should be the case, for the opening a communication between two vessels containing any fluid under the same pressure calls into action no forces but what were previously in action, and therefore the pressure cannot be affected by the diffusion.

Let $V = V'$, then,

$$\text{the elastic force of the mixture} = \tfrac{1}{2}(p + p').$$

When the equilibrium is once established, and the mixture is complete, it is quite immaterial whether the

particles repel all other particles or not. For if they do, since action and re-action are equal, the pressure on the sides of the vessel is the same on either hypothesis.

Thus, in any mixture of gases, as in our compound atmosphere, each component occupies the whole space allotted to them all*; and the absence or diminution of one portion of the mixture produces no effect on the equilibrium of the rest.

96. Prop. *To determine the law of gaseous diffusion on the hypothesis of Dalton's law.*

If the particles of one gas act only on those of its own kind, and consequently one gas be as a vacuum with respect to another, the rate of diffusion of one gas into another must at the beginning of the motion be very nearly the same as the velocity with which a gas rushes into a vacuum. Now the velocity of a gas rushing into a vacuum is known, by analysis, to be inversely as the square root of its density. Hence if v and v' be the rate of diffusion of two gases whose density is ρ and ρ', we have

$$v : v' :: \frac{1}{\sqrt{\rho}} : \frac{1}{\sqrt{\rho'}} \text{ .... (1).}$$

Let V and V' be those volumes of the gases which have at any instant interchanged vessels, then since the volume diffused will be as the rate of diffusion,

$$V : V' :: v : v' \text{ .... (2).}$$

Let m and m' be the quantity or mass of the gases contained in the volumes V and V' which have become diffused through each other, then

$$\frac{m}{m'} = \frac{V\rho}{V'\rho'} = \frac{v}{v'} \times \frac{v'^2}{v^2} = \frac{v'}{v} \text{ .... (3);}$$

* Dalton, *Memoirs of the Manchester Society*, Vol. v.

substituting from (1) and (2);

$$\therefore\ mv = m'v',$$

or the momentum generated in a given time is the same for all gases, that is, the moving force of each moving current is the same*.

Now in the case of a gas diffusing itself through a porous medium, the circumstances are not exactly the same as when a gas rushes into a vacuum. But it seems probable that a retardation of the gas will be occasioned by the obstruction of the medium. But action and re-action being equal, and the momenta generated in the issuing currents being equal, each current will suffer equal loss of motion; hence the remaining momenta will still be equal. If the momenta are equal, the equations (3) (2) (1) obtain, that is, the velocity of the issuing currents, or the rate of diffusion, is inversely proportional to the square roots of the densities of the gases.

From the experiments of Prof. Graham, before alluded to (Art. 93), it appears that the initial rates of diffusion through a porous substance are very exactly in this proportion. And his other conclusion, that the final volumes interchanged are in the same proportion, may be shewn to be included under Dalton's law†.

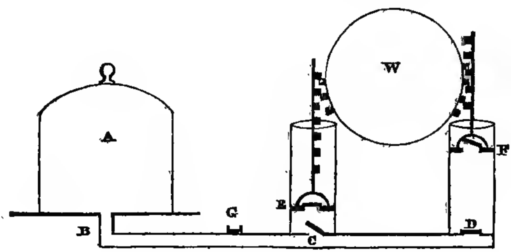
The Common or Double-Barrel Air Pump.

97. The Air Pump is a machine for exhausting the air from any vessel, and its action depends on the dilatability or expansive power of the air (Art. 5): for in virtue of this property, when a portion of the air is withdrawn from any vessel, the remaining quantity immediately dilates and fills the whole space.

* Mr Thomson, *Lond. and Edin. Phil. Mag.* May 1834.

† *Ibid.* See Kelland's *Theory of Heat*, Chap. IV.

A glass vessel *A*, called the receiver, is set on a metal plate which is ground accurately true and polished, so that the edge of the receiver may be in air-tight contact with the surface of the plate; this may always be insured by smearing the edge of the receiver with a little grease. A pipe communicates with the receiver by means of an orifice *B* in the metal plate, and with two cylindrical barrels by valves *C* and *D* opening upwards. In these barrels air-tight pistons *E* and *F*, furnished with valves opening upwards, are worked up and down by the rack-wheel *W*, which is turned by a winch in the usual manner, and is so placed that as one piston ascends the other descends. Thus in the figure, *E* is ascending and *F* is descending, and when *E* is at the top of the barrel, *F* will



be at the bottom; and then the wheel being turned in the opposite direction, *F* will ascend and *E* descend. Now it will be seen that all the valves open upwards, and that when the valve at the bottom of the cylinder is open, the valve in the piston which works in that cylinder is shut, and *vice versâ*. Thus *C* is open and the valve in *E* is shut; and *D* is shut and the valve in *F* is open. Now as the piston *E* ascends from the bottom to the top of the barrel, a partial vacuum is created below the piston, the external air being unable to insinuate itself between the piston and cylinder. The pressure of the air therefore being removed

from the upper surface of the valve C, the elastic force of the air pressing on the under surface of the valve opens it, and the air in the receiver and pipes dilates itself and fills the barrel.

Now during the ascent of E and consequent descent of F, the valve in F is open and the valve D is shut. For as the piston F descends, the air below the piston will evidently shut D and open F. Thus no fresh air can enter the tube or receiver, and by this operation a quantity of air is abstracted from the receiver and tube equal to the content of one of the barrels. Now let the wheel be turned back, and F begin to ascend, the valve in F is immediately closed and D is opened; also E begins to descend, the valve C is closed, and the valve in E is opened, and then another quantity of air equal to the content of one barrel is withdrawn from the receiver. This process may be continued, a barrel of air being expelled each turn of the wheel, till the elastic force of the remaining air is too slight to open the valves, when the action of the pump must cease, since no greater degree of exhaustion can then be obtained.

98. Prop. *To find the density of the air in the receiver after any number of turns of the wheel.*

Let A be the content of the receiver and pipe, and B the content of each of the barrels.

Let ρ be the original density of the air in the receiver, and $\rho_1, \rho_2, \ldots \rho_n$ the density after $1, 2, \ldots n$ turns of the wheel. Then the air which occupied the space A when E was at the bottom of the barrel, expands itself into the barrel as E ascends, and (at the completion of the turn, when E is at the top of the barrel) occupies the space $A+B$. And the quantity of air whose density is ρ_1 is the same as the quantity whose density was ρ:

$$\therefore \rho_1 (A+B) = \rho A \ldots \ldots\ldots\ldots (1).$$

After the second turn when F has ascended to the top of the barrel $\rho_2(A+B)=\rho_1 A$, and so on:

$$\therefore \rho_2=\rho_1 \cdot \frac{A}{A+B} \text{ and } \rho_1=\rho\frac{A}{A+B} \text{ by (1);}$$

$$\therefore \rho_2=\rho \cdot \left(\frac{A}{A+B}\right)^2;$$

and similarly after n turns the density is

$$\rho_n=\rho\left(\frac{A}{A+B}\right)^n=\frac{\rho}{\left(1+\frac{B}{A}\right)^n}.$$

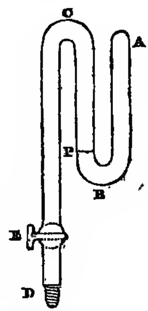
Thus it appears that the density decreases in a geometric progression whose common ratio is $\frac{A}{A+B}$. It appears from this, that the air in the receiver can never be completely exhausted, since the ratio $\frac{A}{A+B}$ always remains finite, and therefore ρ_n has always a sensible value.

99. *Practical limits to the degree of exhaustion.*

From the last article, it appears theoretically impossible to exhaust all the air, or even to obtain a very great degree of rarefaction; the practical difficulties are very great. It is impossible to make the pistons touch accurately the bottoms of the barrels; hence when the pistons are at the bottom, there will be some air left below them, and this will not be of sufficient elasticity to open the valves in the pistons. For as the pistons descend, their valves cannot open until the air below them is of greater elastic force than the atmospheric air, and when the exhaustion has proceeded to a certain extent, a whole barrel of air may be compressed, by the descent of the piston, into the small space between the bottom of the piston and of the barrel, without acquiring sufficient elastic force to open the valve. When this is the case, the rarefaction can proceed no farther, for the

valves at the bottom of the barrels will not open when the piston is at the top of the barrel. For no air having been expelled by the ascent of the piston, since none escaped through the valve when the piston was at the bottom, the air in the pipe and receiver is of the same density, and therefore of the same elastic force as the air in the barrel, and therefore the valves ceasing to open, the action of the pump is entirely suspended.

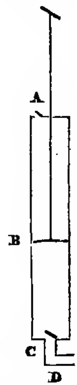
100. *Siphon Gauge.* In using the air-pump, it is desirable to know the degree of exhaustion which exists. This is most conveniently effected by the Siphon Gauge, which is a small glass tube bent into the form represented in the figure, and having the portion *ABP* filled with mercury. There is a stop-cock at *E*, and the extremity *D* can be screwed into some part of the air-pump so as to communicate with the receiver. In the figure of the air-pump (Art. 98), there is an orifice at *G* into which the gauge may be fitted. When the cock at *E* is open, there will be a free communication between the air in the receiver and the surface of the mercury. Now the mercury will not sink in *AB* and rise in *BC*, unless the elastic force of the air on the surface of the mercury be diminished. Hence as the air is withdrawn from the receiver by the working of the pump, the elastic force of the air on the surface of the mercury at *P* will be diminished, and the mercury will rise in *BC* and sink in *AB*. And if the exhaustion were complete, the mercury would stand at the same height in both tubes. But as all the air cannot be withdrawn, the *degree* of exhaustion is accurately measured by the distance to which the mercury sinks in the tube *AB*. And the difference of the levels of the mercury in *AB* and *BC*, is an accurate



measure of the elastic force of the air which remains in the receiver.

Smeaton's or the Single-barrel Air-pump.

101. The single-barrelled Air-pump, which is represented in the accompanying figure, is capable of producing a greater degree of exhaustion than the common air-pump. The top of the barrel is closed, and has a valve in it at *A* opening upwards, which being closed prevents the external air from pressing on the surface of the piston.



Let us suppose that the piston is beginning to be drawn up from the bottom to the top, then the pressure of the air being removed from the upper surface of the valve at *C*, the elastic force of the air in the pipe *CD* leading to the receiver opens the valve, and the lower part of the barrel fills with air as the piston ascends. The valve in *B* is shut, being pressed by the air above the piston, and the valve *A* being pressed on its upper surface by the whole elastic force of the external air, cannot open until the elastic force of the air beneath it becomes by compression greater than the elastic force of the external air.

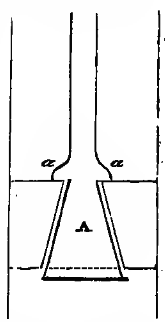
When the piston begins to descend, the valve *A* closes, and the valve in *B* opens, and the valve *C* closes. Smeaton's principal improvement was in the valves, which ought to open on the slightest pressure, or the rarefaction will be suspended. He made the valves of small pieces of oiled silk laid over six holes pierced close together. When the air presses on the upper surface of the valves, as on the valves *A* and *C* during the descent of the piston, and on the valve in *B* during its ascent, the silk is pressed close to the edges of the holes, and the passage of the air is completely prevented; and when the air presses on the under side of the valves, so great

a surface of silk is exposed, owing to the number of the holes, that an exceedingly small difference in the elastic force on the under and upper surface is sufficient just to raise the silk and give a free passage to the air. Unless the upper surface of the piston can be brought into very close contact with the top of the barrel, the valve A will not open; and again, unless its lower surface can be brought likewise into close contact with the bottom, its valve will not open; and the difficulty of effecting this is a practical limit to the exhaustion. The labour of working this pump is less than the labour of working the common air-pump, since the pressure of the external air is cut off by the valve A from the surface of the piston, and only acts towards the end of the stroke when the valve is open.

The preceding shews the construction and working of the Stomach-pump, which is a single-barrel pump with the same set of valves as in this air-pump.

Cuthbertson's Air-pump.

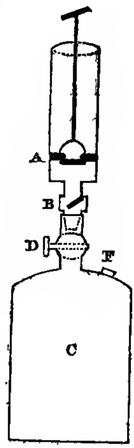
102. In this pump the practical difficulty of bringing the lower surface of the piston close to the bottom of the barrel, so that the valve in the piston may always be opened, is avoided by the ingenious construction of the piston and piston-rod, which is represented in the accompanying figure. The piston is pierced by a conical hole, into which the lower part of the piston-rod, which is a conical plug, fits accurately. When the piston-rod is drawn up, the plug A stops the hole and draws the piston up with it, and the surfaces of the plug and conical hole being in air-tight contact, all the air above the piston is expelled through a valve in the



top of the barrel. When the descent is to be made, the piston-rod being pushed down, the conical plug is detached, and the surfaces of the conical plug and hole are no longer in contact, but being separated, as is represented in the figure, a free passage is given to the air from the lower to the upper side of the piston, and the shoulders *a*, *a* catching on the piston carry it down to the bottom of the barrel.

The Condenser.

103. By means of this instrument a large quantity of a compressible fluid is forced into a very small space.



It consists of a cylindrical barrel, with a valve at the bottom, opening outwards, and an air-tight piston, with a valve opening towards the bottom of the barrel. The piston may be worked up and down either by the hand or by machinery attached to the piston-rod. When it is required to force a quantity of air or gas into a strong vessel *C*, the vessel must have a projecting neck or tube, which can be made to communicate with the condenser, and which is furnished with a stop-cock, or valve opening inwards. Then when the piston *A* is depressed from the top to the bottom of the barrel, the air which was contained in the barrel is forced through the valve at *B* into the vessel *C*; for the valve in *A* is shut, and the valve *B* is opened by the pressure of the air above it. The piston is then raised again, when the valve *B* shuts and the valve in *A* opens, and the barrel again becomes full of air, which is again forced into the vessel *C* by the descent of the piston; and this process is continued till a sufficient quantity has been forced in.

104. *The density of the air in the receiving vessel increases in arithmetic progression.*

Let V be the content of the receiver, and V' of the barrel, and ρ the original density of the air which is forced in. Then ρV is the quantity of air contained in the receiver at first, and $\rho V'$ is the quantity forced in each descent of the piston. Hence, after one descent, the quantity of air in the receiver is

$$\rho V + \rho V' = \rho (V + V').$$

After two descents, it is

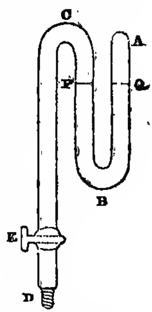
$$\rho V + 2\rho V' = \rho (V + 2V'),$$

and after n descents, it is

$$\rho V + n\rho V' = \rho (V + nV'),$$

or it increases in an arithmetical progression.

105. *Siphon Gauge.* The elastic force of the air in the receiving vessel of the condenser may be measured by a gauge. A glass tube is bent into the form represented in the figure. A portion PBQ of the branches AB, BC is filled with mercury, and the end A is closed. Then the elastic force of the air in AQ on the surface Q of the mercury is the same as the elastic force of the air on the surface P of the mercury. If the pressure of the air on the surface P increase, the mercury will sink in PB and rise in QBA, and the air in AQ will consequently be condensed. And the degree of condensation of the air in AQ will shew accurately the degree of increase of elastic force on the surface P of the mercury.



Let now the gauge be attached by the screw D to the receiving vessel of the condenser (as at F, in fig. Art. 103). Then the stop-cock at E being open, there will be a free communication between the air in the receiver and the surface P of the mercury. And the elastic force on the surface of the mercury will be the same as the elastic force on any portion

of the surface of the vessel equal to the surface of the mercury. Hence the surface P will sink and Q will rise, and the degree of condensation may be accurately measured.

Cor. 1. The amount of pressure to which a vessel is subjected when water is forced into it, as in the experiment (Art. 17), of the compressibility of water, may be measured in this manner.

Cor. 2. The distance AQ decreases in harmonic progression. For the density of the air is inversely as the space it occupies, and the density increases in arithmetical progression. Hence the distance AQ being inversely as the density is in an harmonic progression, for the reciprocals of quantities in harmonic are in arithmetic progression.

The Air-gun.

106. The Air-gun is an instrument for projecting balls or other missiles by the elastic force of condensed air. The elastic force of air in a given space increases as the density (Art. 90); hence, if a large quantity of air be forced by means of the condenser into a strong metal receiver, the elastic force of the air in this receiver may become very great. A communication can be made between the receiver and a gun-barrel, and a ball being placed in the breech of this barrel, and fitting it nearly air-tight, is exposed to the elastic force of this condensed air the instant the communication is opened, and driven out of the barrel as by the expansive force of gunpowder. The force of projection obviously depends on the degree of condensation of the air in the receiver.

The schoolboy's *Pop-gun* is a familiar instance of the action of the elastic force of condensed air.

The Diving Bell.

107. If any vessel, as a large bell or chest, be inverted over water, the air will occupy the upper part of the chest, and diminish in bulk in proportion to the depth to which

it is sunk. At a depth of about 34 feet the air will be diminished by one half its bulk; for the pressure of a column of water of this height is nearly equal to the elastic force of the air at the surface of the earth, and consequently the air in the vessel sustaining double the pressure which it did before the chest was sunk, will occupy half the space.

Now it is necessary to keep the bell clear of water, so as to allow sufficient room for the workmen; and this may be done either by making it very deep, or, as is more convenient, by forcing more air into it by a condenser. The condenser is placed at the surface of the earth, and communicates with the interior of the bell by a long flexible pipe. Then as the air is forced in, the elastic force increasing with the condensation, keeps the bell clear of water. In a short time the air in the diving bell would become unfit for respiration, and the supply of fresh air which is forced in by the condenser not only serves to keep the bell clear of water, but is absolutely necessary for the existence of the diver. As fresh air is forced in, the impure air may be let out by a stop-cock; or, when the bell is quite clear of water, a portion will escape into the surrounding water at each stroke of the condenser; and, rising through the water in bubbles, escape at the surface.

The diving bell receives its name from the shape originally given to it; it is now generally constructed as a deep oblong chest, narrower at the top than at the bottom.

108. PROP. *To determine the space clear of water in a diving bell sunk to a given depth.*

Let V be the content of the bell, and h the height of a column of water whose pressure is equal to that of the atmosphere, that is, to the elastic force of the air in its natural state.

Let the top of the bell be sunk to a given depth h' below the surface of the fluid, and let V' be the content of the part of the bell which is clear of water.

Let x be the depth of the surface of the water below the

top of the chest. Then the pressure at any point of the surface of the water in the bell is the weight of a column $h+h'+x$, being as the depth below the surface of the water; and the elastic force of the air being inversely as the space occupied, we have

$$h : h+h'+x :: \frac{1}{V} : \frac{1}{V'};$$

$$\therefore\ V' = \frac{Vh}{h+h'+x},$$

a general expression whence the space may be found.

Ex. 1. Let the bell be a prismatic chest, whose height is a. Then

$$V' : V :: x : a;$$

$$\therefore\ \frac{V'}{V} = \frac{x}{a} = \frac{h}{h+h'+x};$$

$$\therefore\ x^2 + (h+h')\,x = h\,a,$$

$$\text{and } x = -\tfrac{1}{2}(h+h') \pm \sqrt{\tfrac{1}{4}(h+h')^2 + h\,a};$$

whence for any given values of h, h', a, the portion clear of water may be found, the positive sign being adopted.

Gas-meters.

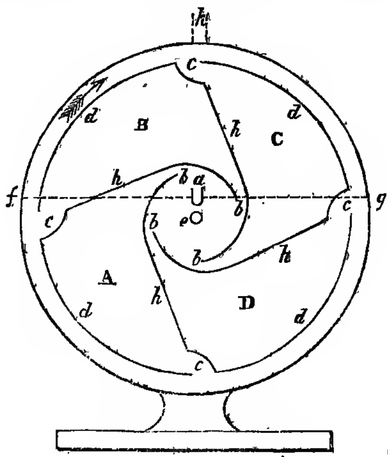
108*a*. The construction and operation of gas-meters afford a good illustration of the laws of elastic fluids and of the action of an elastic fluid on a solid and a liquid. Gas-meters are of two descriptions, called technically, the *wet* and the *dry* meter. The object of both is the same, namely, to measure the quantity of gas used by the consumer and supplied by the manufacturer.

Wet Gas-meters. The wet gas-meter derives its name from the fact, that water is employed in the measuring of gas, and is essential to the action of the apparatus. This description of meter* consists of one hollow drum or cylinder, revolv-

* This very beautiful and ingenious apparatus was the invention of Mr Clegg, who had a patent for it in 1815. See Webster's *Patent Cases*, pp. 103—106.

ing about a horizontal axis within another hollow drum or cylinder, the inner drum being divided into compartments for measuring the gas, and revolving in water which occupies the lower part of the outer cylinder to a height above the axis, about which the inner drum revolves. The details of the construction may be varied, but the wet gas-meters have this common feature, that the gas is measured in the compartments of a drum, which compartments are occupied successively by gas and water, the drum being made to revolve by the pressure or elastic force of the gas acting on the compartments of the drum in succession. The revolutions of the drum being registered by suitable apparatus, the quantity of gas which has passed through the meter by so filling the compartments in succession, will be accurately measured.

In the annexed figure, the outer circle represents the outer case or drum of the meter, within which a drum divided into compartments *A*, *B*, *C*, *D* revolves about an horizontal axis *e*. The gas to be measured is brought into the meter by a pipe passing horizontally in the direction of the axis of the inner and outer cylinder, and turned up at the end so that its orifice



a may stand above the water-level *fg*. The four compartments *A*, *B*, *C*, *D*, are similar in every respect, each having an inlet *b* by which the gas enters the compartment, and an outlet *c* by which it passes out of the compartment into the upper part of the outer case, whence it may pass by a pipe *k* in any convenient direction. The gas being admitted into the meter, will pass the inlet *b* into the part of the compartment *A* which is just rising out of the water; the gas presses equally on the surface of the water and on the side *h* of the compartment *A*; the effect of this pressure on the side *h* is to cause the inner drum to revolve, whereby the compartment *A* is raised more and more out of the water, and as it rises, it fills with gas until it occupies, by the revolution of the drum, the position in space of the compartment *B*, as to which it will be observed that the inlet *b* has just dipped below the surface of the water, and the outlet *c* is just coming to the surface of the water. The outlet *c* having risen above the surface, the gas will escape into the upper portion of the outer case. As the gas passes out of a compartment in the situation of *B*, by the outlet *c*, water will enter by the inlet *b*, and as the drum revolves the compartment *A* having occupied the position in space of *B* and *C* comes into the position *D*, when it is entirely emptied of gas and filled with water, until by further revolution of the drum, the compartment having come again into the position of *A* begins to fill with gas, as already described. That which has been described for one compartment takes place for the other three; and it will be seen that there will always be two compartments discharging their gas into the outer case above the water-level, one compartment filling with gas, and as it fills causing the inner drum to revolve, and one compartment full of water. The motive power causing the inner drum to revolve, is the pressure of the gas in the mains as transmitted from the gasometer at the gas-works (Art. 127), and the quantity of gas will be ascertained from the number

of compartments which have filled and emptied, that is, from the number of revolutions of the inner drum, which are registered by a train of wheel-work and a dial-plate in the usual manner. The gas so measured and passed through the meter is conveyed to the lights by a pipe communicating in any convenient manner with the outer case, and the action of the meter is suspended when the gas does not pass away. It will be observed, that the quantity of gas measured will be affected by the height of the water in the meter, the portion of the compartment which is occupied by gas being greater or less, according to the level of the water, whereas the inner drum will revolve whatever the height of the water within certain limits. If the water rises above the top of the supply pipe *a*, the entrance of the gas may be stopped altogether, and if it sinks so low that the orifices *b* and *c* are not sealed or closed and opened simultaneously, the gas may pass through the meter without causing the inner drum to revolve. Hence every gas-meter is or ought to be adjusted to a certain water-level; if the water rise above this level each compartment will measure too little gas, and since the number of revolutions is the same whatever the available space in the compartment for holding gas, the consumer will be defrauded; on the other hand, if the water sink below this level, since the available space in each compartment is larger than was calculated on, the manufacturer will be defrauded. Various causes are in operation to occasion a variation in the water-level, and the necessity of preserving it has given rise to several ingenious contrivances for discharging any excess or supplying any deficiency of water; upon the details of which, however, it would be foreign to our purpose to enter*.

Dry Gas-meters. The inconveniences attendant on the wet gas-meters, from the difficulty of maintaining a permanent water-level, led to the invention by Mr Defries of the

* See On the Manufacture of Gas, by Samuel Clegg, jun., 4to.

dry meter, consisting in its simplest form of two chambers, so contrived as to expand or dilate on the gas entering from the gas-holder and to contract on the gas passing out to the consumer, the alteration in the capacity of the chambers as they fill and empty alternately being effected by a moveable diaphragm of flexible materials, or of plates fastened together and connected with an apparatus whereby the number of chambers filled and emptied may be registered. In this meter also the elastic force or pressure of the gas is the motive power, and various modifications and arrangements of a diaphragm have been adopted for the purpose of reducing and obviating its rigidity and resistance to motion.

CHAPTER VIII.

ON THE ATMOSPHERE.

109. In the instances presented to our notice by nature, space is occupied by a material substance, for the moment any substance is removed, the surrounding air rushes in and fills its place. This circumstance gave rise to the ancient physical dogma, that nature abhors a vacuum, which seemed to explain the cause of a supposed law in physics, that it is impossible for space to exist unoccupied by matter. Suction by the mouth affords a ready illustration, for if the air be withdrawn from a tube, one end of which is immersed in water, the water succeeds immediately in the tube as the air is withdrawn. The fact is certain, and the explanation of the phenomena was, that because nature abhors a vacuum, she compels the water to occupy the space deserted by the air. The rise of the water on raising the piston of the suction-pump was explained in the same manner. The suspension of fluid in tubes inverted with open ends immersed in a vessel containing the same fluid was explained in the same manner; and the natural gravitation of the suspended column on the fluid in the vessel was supposed to be counteracted by the adhesion of the upper surface of the column to the top of the tube; and nature's horror would not permit a separation between these two surfaces, except in the case of a column of mercury of more than 29 inches, which was sufficient to take off layers of mercury, and thereby make a chain to let down the column to this height above the surface of the mercury over which it was inverted*.

* See Cotes's *Hydrostatics*, Lecture VII.

110. *Discoveries of Torricelli and Pascal.* But an attempt being made to raise water from a depth of more than 32 feet, it appeared that nature's horror here too had limits, and only extended to 32 feet, and that her disinclination to an empty space did not extend beyond this.

From this circumstance Torricelli argued that whatever be the power which sustains the column of water, its energy must be measured by the weight of the column; and consequently, that if a heavier fluid be used, a much less column would be sustained. He tried mercury, which is nearly 14 times heavier than water, and the consequence was, that the column sustained in this case was about $\frac{1}{14}$th the column of water, or about 28 inches. The identity of the cause which operates in the two cases was thus satisfactorily established, and the cessation of the gravitation of the fluid being most improbable, Torricelli sought the true cause in the pressure exerted on the surface of the fluid exposed to the atmosphere.

If the pressure of the atmosphere is the true cause of these phenomena, a less column of fluid would be sustained when the pressure is less. And this pressure being the weight of the superincumbent column (Cor. Art. 43) would be less as the column is less; that is, the pressure would be less at the top of a mountain or building than at the bottom, and a less column ought to be sustained at the upper than at the lower stations. The experiments of Torricelli and Pascal shewed clearly that this is the case.

111. Prop. *To make a Barometer.*

Experiments on this subject are usually made with a barometer, which is constructed in the following manner. A small glass tube of about 32 inches in length, with one end hermetically closed and the other open, is filled with mercury, and a finger being placed over the open end, is

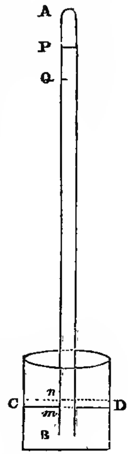
inverted in a vessel of mercury called the cistern or basin, and the finger being removed, the mercury in the tube communicates with the mercury in the basin, and will stand at a height varying from 28 to 31 inches above the mercury in the basin. The internal diameter of the tube should not be less than one-eighth of an inch. The mercury should be perfectly dry and pure, and the tube also perfectly dry; for if there be any moisture it will expand into an elastic vapour when the pressure of the air is removed, which ascending into the vacuum at the top of the tube will depress the mercury. The moisture and air may be expelled from the mercury by boiling it in the tube. When a barometer is well filled, the space at the top is the most perfect vacuum with which we are acquainted.

112. Prop. *The mercury is sustained in the barometer by the pressure of the air upon the surface of the mercury in the basin, and the pressure of the atmosphere is accurately measured by the height of this column.*

Let the barometer be placed under the receiver of an air-pump, then as the air is exhausted the mercury sinks in the tube; and if all the air could be exhausted, so that no pressure would be exerted on the surface of the mercury in the basin, the surfaces of the mercury in the tube and in the basin would coincide. When the air is admitted again into the receiver, the mercury rises in the tube to its former height.

The pressure of a fluid on any portion of the surface is the weight of the superincumbent column of the fluid (Art. 43). And here the pressure of the mercury in the tube AB on the surface CmD of the mercury in the basin is the weight of the superincumbent column Pm, if P be the surface of the mercury in the tube. And this pressure is balanced by the pressure of the air on the surface of the mercury in the basin.

Now when a fluid is at rest, the pressure on all equal portions of the surface is the same. (Cor. Art. 31.) Hence the pressure of the air on any portion of the surface of the mercury in the basin equal to the section of the column sustained in the tube, is equal to the pressure of this column. But these pressures are the weights of the superincumbent columns.



And the weight of a superincumbent column whose section is unity is (Art. 33)

$$p = g\rho z.$$

Hence if the section of the barometric column be equal to the unit of area, the weight of the atmosphere, being equal to the weight of this column, is $g\rho z$.

Cor. The weight of the atmosphere is $g\rho z$ where ρ and z refer to the barometric or mercurial column. But the same expression would give the weight of the atmosphere if ρ and z referred to the atmospheric column. But these quantities evidently cannot be determined directly for the atmosphere.

113. Prop. *To correct the barometric column for the rise and fall of the surface of the mercury in the cistern.*

As the surface P of the column sinks, the surface CmD rises. Let Q and n be other corresponding positions of the two surfaces. Thus the two surfaces move always together but in opposite directions; and PQ is the apparent variation of the barometric column, but the real variation is $PQ + mn$. Now if the area of the cistern be very large, as compared with the area of the tube, the quantity mn will be insensible, and the apparent variation may be taken for the real one without sensible error.

It is however very inconvenient to have a large cistern; and the quantity mn by which PQ is to be increased, may be readily calculated for every value of PQ, and marked on the barometer. The calculation is as follows. Let K be the horizontal section of the cistern, and k of the tube, then $k \cdot PQ = K \cdot mn$;

$$\therefore\ mn = \frac{k}{K} PQ$$

the real variation

$$= PQ + mn = PQ + \frac{k}{K} PQ$$

$$= \left(1 + \frac{k}{K}\right) PQ.$$

The ratio $\frac{k}{K}$ is constant in the same barometer; and the real variation is therefore known for every value of PQ.

If m be the zero point of the scale of inches, the apparent height of the mercury is Qm.

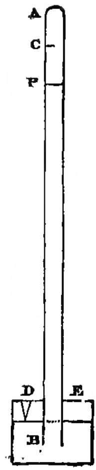
But the true altitude $= Qn$

$$= Pm - (PQ + mn)$$

$$= Pm - \left(1 + \frac{k}{K}\right) PQ.$$

The correction $\frac{k}{K} PQ$ is called the correction for the relative capacity of the tube and cistern, and its value is marked on most barometers.

114. *True barometric column.* If the surface of the mercury in the cistern is always at the same height, the true altitude and variation of the barometric column will be given at once by the scale of inches, without any correction for the relative capacity. This may be effected by making the cistern with a moveable bottom, which fits accurately,



and may be elevated or depressed by means of a screw, according to the rise or fall of the barometric column; so that the mercury in the cistern being elevated or depressed, its surface may always meet the point of an index *D* attached to the cistern. The extremity of this index is in the same horizontal plane as the zero of the scale of inches, when the tube *AB* is vertical. Thus the scale of inches gives at once the height of the mercury in the tube above the mercury in the cistern, that is, the true barometric column. The tube *AB*, except a small opening between the 28th and 31st inches of the tube opposite which the graduation is placed, is enclosed in a strong case and firmly attached to the cistern, and a very small hole is left at some point *E* for the action of the atmosphere on the surface of the mercury in the cistern*.

115. Prop. *To find the altitude of the mercury in the barometer when a portion of air has been allowed to remain in the tube.*

In order that the barometric column may indicate the exact weight of the atmospheric column, there must be no air or vapour left above the surface of the mercury in the tube (Art. 111); for if any is left there, it will exert pressure on the surface of the mercury and assist in balancing the atmospheric column, so that the barometric column will not stand so high as it ought to do: the inaccuracy arising from this cause may be allowed for in the following manner.

Let as much air be left in the tube as in the natural state of the atmosphere would occupy the space *AP* (fig. Art. 112),

* Various ingenious expedients are adopted by different instrument-makers to adjust the surface of the mercury with convenience and accuracy. See Chap. XVI. as to the effect of Capillary Attraction.

and when the tube is inverted over mercury, let the barometric column stand at Q, then the air expands itself and occupies the whole space AQ.

Let a be the height of the end A of the tube above the surface of the mercury in the cistern, and x the height of the surface Q and $AP = b$.

Let h be the height of the true barometric column, that is, of the column of mercury which would be sustained by the weight of the atmosphere, or by the elastic force of the air when contained in the space AP.

Now Q being the position of the surface of the sustained column, we have

weight of the atmosphere = weight of column Qm + elastic force of air in AQ.

But the weight of the atmosphere $= g\rho h$, and the weight of the column $Qm = g\rho x$; substituting then in preceding

$$g\rho h = g\rho x + \text{elastic force of air in } AQ;$$

$$\therefore \text{ elastic force of air in } AQ = g\rho(h - x).$$

Elastic force of air in AQ : elastic force in AP :: $\dfrac{1}{AQ} : \dfrac{1}{AP}$,

$$\text{or } g\rho\,(h - x) : g\rho h :: \frac{1}{a - x} : \frac{1}{b};$$

$$\therefore (h - x)(a - x) = hb \quad \ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Solving the equation

$$x = \tfrac{1}{2}(a + h) \pm \tfrac{1}{2}\sqrt{(a + h)^2 - 4h\,(a - b)} \quad \ldots\ldots\ldots\ldots\ldots(2).$$

COR. 1. The values of b in the preceding proposition must lie betwixt zero and a.

Let $b = 0$; then

$$x = \tfrac{1}{2}(a + h) \pm \tfrac{1}{2}(a - h)$$
$$= h \text{ or } a,$$

the former of which is the case of a perfect barometer, the latter inadmissible.

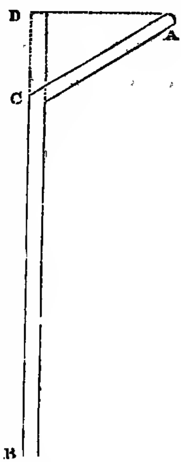
Let $b = a$; then $x = 0$, or $(a + h)$, in the former case the mercury does not rise in the tube; the latter is inadmissible.

When b has intermediate values betwixt 0 and a the upper sign must be used, as will be seen in any solution.

Cor. 2. The preceding equation may be employed to ascertain any one of the quantities, the other three being given; it will generally, however, be applied to correct the apparent barometric column for a given quantity of air or vapour left above the mercury, or to ascertain the quantity of air for a given depression.

Diagonal Barometer.

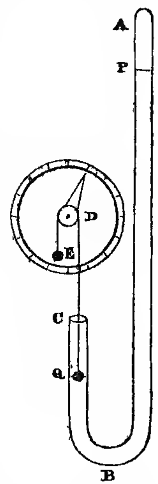
116. The entire variation of the mercury does not in general exceed 3 inches, hence very small variations in the state of the atmosphere are with difficulty observed in a vertical barometer. Various contrivances have been suggested for increasing the range of the mercury in the tube, and thereby rendering the least change perceptible. One contrivance for effecting this consists in bending the upper 3 or 4 inches of a barometer tube into an oblique position, as represented in the figure. The variation which in the common vertical barometer takes place through the space CD, has a greater range in the diagonal arm CA, and any change in the height of the mercury will be increased in the ratio of the length of CA to the length of CD.



Wheel Barometer.

117. The contrivance most frequently used for enlarging the scale is represented in the accompanying figure, and

known by the name of the Wheel Barometer. A common barometer tube is bent so that AB and BC may be parallel, and AB about 31 inches longer than BC. The tube AB is filled with mercury, which will rise in BC until the difference of the levels of P and Q equals the height of the column, which is equal to the pressure of the atmosphere. On the surface of the mercury in BC floats an iron ball to which a string is attached, and carried round a wheel at D, having at its extremity a weight E nearly equal to the iron ball. The iron ball then just floats on the surface Q of the mercury, and as it ascends or descends the wheel D revolves and carries a hand or index pointing to the divisions of a graduated plate. The section of the legs AB and BC being of equal area, the change in the height of the barometric column is double the change of the surface Q.



Now the barometric column changes through about 3 inches; hence the change of the surface Q will be about $1\frac{1}{2}$ inch. If then the circumference of the wheel D be $1\frac{1}{2}$ inch, the hand will be carried completely round the dial-plate by the ordinary changes of the barometric column in this climate. The dial-plate may be any size, so that the slightest variation will be perceptible. Suppose the rim of the plate to be divided into 300 equal parts, then the motion of the index over one of these parts will indicate a change in the surface of the mercury equal to a 300th part of its whole change, that is, equal to $\frac{1}{300}$th of $1\frac{1}{2}$ inch, that is, a 200th part of an inch. And the real change in the barometric column is double this, or a 100th part of an inch. The extremely small variation then of the one hundredth part of an inch in the barometric column is readily exhibited by a wheel barometer,

But there are sources of inaccuracy arising from the friction of the wheel, the humidity of the string, and the weight of the iron ball, which render this instrument unfit for scientific observations, and less valuable than the simple barometer.

118. *On the Weather.* The wheel barometer is in very general use as a Weather-glass. The barometer column is observed to fluctuate in connexion with changes in the state of the weather; hence rules are attempted to be established by which the coming state of the weather may be predicted from the height of the barometric column; and the words "Fair," "Change," &c. are very commonly engraved on the wheel barometers, and sometimes on vertical ones. But nothing can be more fallacious than any attempt to predict the state of the weather from the absolute height of the mercury, since the absolute height differs for every place on a different level (Art. 110); and a good barometer used thus, as a weather-glass, would indicate different states of weather at the bottom and top of a lofty house. But the state of the weather may be predicted from a change in the height of the barometric column; and it may be observed, that when the mercury is rising its surface is convex, and when falling it is concave.

It appears, according to the observations of Dalton, that the highest mean of the barometer upon six successive months is from March to August inclusive, and that the lowest mean of rain is during the same six months; thus the quantity of rain and the height of the barometer are just the reverse of each other, the one being least when the other is greatest.

It appears also, that the higher the barometer is above its mean annual state, the less rain there is; and the lower it is below its mean annual state, the more rain there is*.

* See Dalton's *Meteorology*, 2nd Edit. Essay VII.; and Howard's *Climate of London.*

The following rules are given by Dalton respecting the weather in connexion with the variations of the barometer.

(1) The barometer is highest of all during a long frost, and generally rises with a N. E. wind. It is lowest of all during a thaw following a long frost, and is often brought down by a S. W. wind.

(2) When the barometer is near the high extreme for the season of the year, there is very little probability of immediate rain.

(3) When the barometer is low for the season, there is seldom a great weight of rain, though a fair day in such a case is rare. The general tenor of the weather at such times is short, heavy, and sudden showers, with squalls of wind from the S.W., W., or N.W.

(4) In summer, after a long continuance of fair weather, with the barometer high, it generally falls gradually, and for one, two or more days before there is much appearance of rain.

If the fall be sudden and great for the season, it will probably be followed by thunder.

(5) When the appearances of the sky are very promising for fair, and the barometer at the same time low, it may be depended upon that the appearances will not continue long. The face of the sky changes very suddenly on such occasions.

(6) Very dark and dense clouds pass over without rain when the barometer is high; whereas when the barometer is low it sometimes rains almost without any appearance of clouds.

(7) All appearances being the same, the higher the barometer the greater the probability of fair weather.

119. Prop. *If the atmosphere be conceived to be divided into a number of parallel strata of equal thickness, the mean density of these strata will diminish in geometrical progression.*

Let the atmosphere be divided into a number of strata of equal thickness and parallel to the surface of the earth, and let the density of each of these strata be considered uniform throughout the strata, that is, to be the mean density. Now the air being perfectly compressible, and the upper parts pressing on the lower (Art 9), the lower strata will be more pressed than the upper, since they sustain a greater superincumbent column; and the density being as the compressing force, will be as the weight of the superincumbent column.

Let $\rho_{,}$, ρ, ρ', be the density of any three contiguous strata.

Then if $p_{,}$, p, p', be the units of pressure on the *upper* surfaces of these three strata respectively, we have (Art. 90)

$$\rho_{,}=\frac{1}{k}p_{,}, \quad \rho=\frac{1}{k}p, \quad \rho'=\frac{1}{k}p';$$

$$\therefore \rho_{,}-\rho=\frac{1}{k}(p_{,}-p),$$

$$\rho-\rho'=\frac{1}{k}(p-p'),$$

$$\rho_{,}-\rho : \rho-\rho' :: p_{,}-p : p-p'.$$

But $p_{,}-p$ is the weight of a column of the second stratum whose section is unity, and $p-p'$ is the weight of a similar column of the third stratum, and these are as the densities of the strata;

$$\therefore p_{,}-p : p-p' :: \rho : \rho';$$

$$\therefore \rho_{,}-\rho : \rho-\rho' :: \rho : \rho';$$

$$\therefore \rho_{,}\rho'-\rho\rho'=\rho^2-\rho\rho';$$

$$\therefore \rho=\sqrt{\rho_{,}\rho'};$$

or the density of the intermediate stratum is a mean proportional between the densities of the other two. Hence it follows that the mean densities of the successive strata

of the atmosphere are in continued proportion; and if the distances be taken in arithmetic progression, the mean densities of the strata will be in geometric progression.

Let unity be the mean density of the stratum next the earth, and r the ratio of the mean density of the next stratum to the density of this, then the mean densities of the successive strata of the atmosphere are

$$1, r, r^2, r^3, \ldots\ldots\ldots$$

120. PROP. *The levels of places may be ascertained by means of the barometer.*

The most important scientific application of the barometer is to the purposes of levelling; that is, to the determination of the relative heights of places above the surface of the earth. For the barometric column being sustained by the atmospheric column, and the altitude of the former being an accurate measure of the weight of the latter, which is evidently less at higher levels than at lower, the diminished height of the barometric column will serve to determine the level of the place.

Let ρ, ρ' be the density of the atmosphere at any altitudes x and x', then since the density of the atmosphere decreases in a geometrical progression, if r be the common ratio of that progression, we shall have

$$\rho : \rho' :: r^x : r^{x'};$$

$$\frac{\rho}{\rho'} = r^{x-x'};$$

then taking the logarithms

$$x - x' = \frac{\log . \frac{\rho}{\rho'}}{\log . r}.$$

But if h and h' are the heights of the barometer at these two places, since the density is as the compressing force, which is as the weights of the barometric column,

$$\frac{\rho}{\rho'} = \frac{h}{h'};$$

$\therefore\ x - x'$, or the difference between the heights of the places, which is the elevation of one place above the other,

$$= \frac{\log . \frac{h}{h'}}{\log . r}.$$

Such is the fundamental principle of the admeasurement of heights by the barometer; but several corrections are requisite to render the operation sufficiently accurate for practice. The most important corrections are those due to the variation of gravity, and the change of temperature at the two stations. Gravity varies inversely as the squares of the distance; hence the weight of the atmosphere varies according to it, and the intensity of gravity is much less at the higher than at the lower station.

Again, the higher station is much colder than the lower, and a correction in the elastic force of the air must be applied on this account, and it will also be necessary to correct for the variation of the bulk of the mercury in the barometer.

The formulæ for the complete solution of this interesting question will be given in the four following articles:

121. Prop. *To find the difference of the altitude of two stations by means of the barometer and thermometer.*

The general formulæ which analysis* furnishes for determining the pressure for a point at a height z above the surface of the earth, g being the gravity at the surface, r the radius of the earth, and $p_{,}$ the value of p when $z = 0$, that is, the pressure at the surface of the earth, is

$$\log \frac{p}{p_{,}} = -\frac{gr}{k(1+\alpha\theta)} \frac{z}{(r+z)} \quad \text{................} \quad (1).$$

To apply this formula to determine the distance of any point above the surface of the earth.

* See *Theory of Fluids*, Art. 82.

Let z' be the height of the upper station, and let p' be the value of p at that point.

Let $\tau_{,}$ be the number of degrees by which the temperature at the surface of the earth exceeds the standard temperature, and τ' the number of degrees for the point at the height z'.

Now the change of temperature as we ascend from the surface of the earth is gradual and nearly uniform for small elevations, hence there will be no great error in assuming the quantity $\theta = \dfrac{\tau_{,} + \tau'}{2}$.

Let $h_{,}$, h' be the observed heights of the barometric column at the lower and upper stations, then since (Art. 112) $p_{,} = mgh_{,}$, $p' = mgh'$,

$$\frac{p'}{p_{,}} = \frac{h'}{h_{,}}.$$

Making then these substitutions in the equation (1) and changing the sign, since

$$\log \frac{p_{,}}{p'} = -\log \frac{p'}{p_{,}}, \text{ we have}$$

$$\log \frac{h_{,}}{h'} = \frac{grz'}{k\left(1 + \alpha\,\dfrac{\tau_{,} + \tau'}{2}\right)(r + z')} \quad \ldots\ldots\ldots\ldots\ldots(2),$$

whence z', the height above the surface of the earth, may be found, since all the other quantities are known, and the height of any other station being ascertained in the same manner, the elevation of one above the other is determined.

122. The preceding equation will require several corrections.

1°. The temperature will be different at the station whose height is required, and at the surface of the earth, and the mercury will be denser at the colder place than at the other, and consequently the same atmospheric pressure sustains a less column than it would have sustained had the

temperature remained unchanged. Hence to compare the pressures at the station and at the surface of the earth, the barometric column must be reduced to the same density; and the column at the colder place must be increased by the quantity by which it would expand at the temperature of the warmer.

Let β be the coefficient expressing the change in bulk which each unit of volume undergoes for each degree of temperature.

Then since $(\tau_{,} - \tau')$ is the difference of temperature of the two places (the upper being taken as the colder), each unit of bulk of the barometric column is diminished by $\beta\,(\tau_{,} - \tau')$; and this correction may be considered as due to the height simply, no correction being necessary for the diameter of the column, since glass and mercury expand and contract equally at ordinary temperatures. Instead therefore of using the observed height h', we must use

$$h'\{1 + \beta\,(\tau_{,} - \tau')\}.$$

Hence $\log h'$ is to be replaced by

$$\begin{aligned}\log h'\{1 + \beta\,(\tau_{,} - \tau')\} &= \log h' + \log\{1 + \beta\,(\tau_{,} - \tau')\}\\ &= \log h' + M\beta\,(\tau_{,} - \tau'),\end{aligned}$$

nearly, where M is the modulus of the system of logarithms.

2°. The force of gravity varies with the latitude, hence g is not constant for all places on the earth's surface; and the general expression for gravity in terms of the latitude is

$$g = E(1 + n\sin^2\lambda)^*,$$

where E is the equatorial gravity, and n a known quantity.

Then if G be the force of gravity at latitude 45°,

$$G = E\left(1 + \frac{n}{2}\right),$$

* *Figure of the Earth.*

$$g = G \frac{1 + n \sin^2 \lambda}{1 + \frac{n}{2}} = G \left\{1 - \frac{n}{2} (1 - 2 \sin^2 \lambda)\right\} \text{ nearly}$$

$$= G \left(1 - \frac{n}{2} \cos 2\lambda\right).$$

3°. The coefficient α will require some correction, and also the constant k.

In determining the values of these quantities, the air was either supposed to be dry, that is, not to contain any aqueous vapour, or that the quantity of that vapour is constant. But as the temperature increases, the quantity of vapour increases also in the atmosphere, and the elastic force of the vapour being added to the elastic force of the air, the increment of volume for a given volume of air must be greater for air which contains vapour than for dry air, hence α must be increased by a small quantity.

For the same reason k will require a small correction, since it expresses the ratio of the elastic force to the density at a given temperature of air that is dry, or contains a constant quantity of vapour.

123. For practical purposes an approximate value of the general equation (2) (Art. 121), may be found. Multiplying up it becomes

$$\frac{rz'}{r + z'} = \frac{k}{g} \left(1 + \alpha \frac{\tau_, + \tau'}{2}\right) \log \frac{h_,}{h'}.$$

$$\text{But } \frac{rz'}{r + z'} = z' \left(1 - \frac{z'}{r}\right)^{-1} = z',$$

very nearly in all cases to which the barometer can generally be applied;

$$\therefore\ z' = \frac{k}{g} \left(1 + \alpha \frac{\tau_, + \tau'}{2}\right) \log \frac{h_,}{h'}$$

$$= \frac{k}{g} M \left(1 + \alpha \frac{\tau_, + \tau'}{2}\right) \log \frac{h_,}{h'},$$

where M is the modulus of the common system of logarithms.

Now $\frac{k}{g}M$ may be taken equal to 20117 yards, and $\frac{\alpha}{2}$ equal to $\frac{1}{900}$, as mean values. Whence

$$z' = 20117\left\{1 + \frac{\tau_{,} + \tau'}{900}\right\}\log\frac{h_{,}}{h'},$$

which will be found a convenient formula for determining in yards the elevation of one station above another, the temperatures being the number of degrees above 32^0 F.

124. Prop. *To find the height of a homogeneous atmosphere.*

Since the upper parts of fluids gravitate on the lower (Art. 9), the lower parts of a compressible fluid, as the atmosphere, must be more dense than the upper, and the air must diminish in density as we ascend, so that the density is a very variable quantity. If the atmosphere were of one uniform density, the height of it would be readily calculated; but as this is not the case, the height of it can only be calculated on some hypothetical law of its density; as for instance, that it is homogeneous, that is, that the density throughout is the same as its density at the surface of the earth.

Let ρ' be this density and z' the height of this atmosphere, then $g\rho'z'$ is the weight of a column whose section is unity; and $g\rho z$ being the weight of the barometric column in equilibrium with this,

$$g\rho'z' = g\rho z; \quad \therefore z' = \frac{\rho}{\rho'}z.$$

Now the mean value of z is 30 inches, and the ratio of the densities being as the specific gravities of the substances, (Art. 77)

$$\frac{\rho}{\rho'} = \frac{13{\cdot}58}{{\cdot}001299} = \frac{1358}{1299} \times 10^4;$$

$$\therefore\ z' = \frac{1358}{1299} \times 10^4 \times 30 \text{ inches}$$

$$= \frac{1358 \times 10^5 \times 3}{1299 \times 36} \text{ yards}$$

$$= \frac{679 \times 10^5}{7794} = 8710 \text{ nearly}$$

$$= \frac{8710}{1760} \text{ miles} = 4{\cdot}9,$$

or the height required is about 5 miles.

125. Prop. *The mean value of the atmospheric pressure in this climate is about* 15lbs. *on the square inch.*

The barometric column which is sustained by the weight of the column of the atmosphere is subject to certain fluctuations, being higher at one time than at another. Hence the weight of the atmosphere is also subject to a like variation, exerting a greater pressure at one time than at another. But in this climate the mean height of the barometer may be taken at 30 inches, and the mean pressure of the atmosphere will be the weight of a column of mercury of this height; and the section of these columns being equal to a square inch, the weight of the barometric column will be the weight of 30 cubic inches of mercury; therefore the atmospheric pressure on a square inch

$= \text{the weight of a cubic inch of mercury} \times 30$

$= 7{\cdot}85 \times 30$ ounces, (Ex. 6. Art. 87)

$= \frac{235{\cdot}5}{16}$ lbs. $= 14{\cdot}9$,

or the pressure on a square inch is about 15lbs.

126. Prop. *The elastic force of any portion of atmospheric air is measured by and in equilibrium with the weight of the atmospheric column and conversely.*

The pressure of the atmosphere at any point may be referred in terms to the elastic force of the air, or to the

weight of the atmospheric column at that point; and although in considering the question of atmospheric pressure, the atmosphere being in a state of equilibrium, it is immaterial whether we speak of the elastic force of the air or the weight of the atmospheric column, it is important that the attention of the student should be directed to the consequences of the laws of elastic fluids as influenced by temperature (Art. 92), when the pressure of the atmosphere is under consideration.

Let any portion of the atmosphere be conceived to be separated from the rest, and inclosed in an expansible envelope, then if the temperature of the air so inclosed be raised, its elastic force will be increased (Art. 91), and the envelope will expand until by the increase of volume and consequent decrease of density (Art. 90), the elastic force of the enclosed air becomes equal to the weight of the atmospheric column. If the temperature of any portion of air be lowered, the elastic force will be diminished, and the volume decrease, until the pressure due to the elastic force of that portion of the air is in equilibrium with the atmospheric column. That which has been just illustrated by supposing a portion of the air to be enclosed in an expansible envelope, is constantly going on in the surrounding atmosphere; the elastic force of the air is subject to continual changes arising from variations in temperature, and its consequent expansion and contraction, until the pressure due to the elastic force becomes equal to and in equilibrium with the pressure due to the weight of the atmospheric column, give rise to the motions of the atmosphere which exhibit themselves in winds and currents. When then the atmosphere is in equilibrium, the elastic force of the air at any point is in equilibrium with and measured by the weight of the superincumbent atmospheric column at that point, and conversely the weight of the superincumbent atmospheric column is measured by the elastic force of the air.

126 *a*. *Aneroid Barometer*. This instrument illustrates the proposition of the equilibrium which may subsist between the elastic force of air and the weight of the superincumbent column of the atmosphere. A vessel, as a flat cylindrical dish, is covered with a flexible diaphragm or plate, so as to be hermetically closed. A portion of the enclosed air being removed by suction from the interior the diaphragm becomes slightly depressed, that is to say, to such an extent as to establish an equilibrium between the weight of the atmospheric column on the exterior and the elastic force of the air in the interior; the amount of this depression will depend on the weight of the atmospheric column, that is, on the state of the weather, and suitable mechanism being provided to measure this depression and exhibit its amount on a dial or any convenient scale, the instrument may be used for like purposes as the mercurial barometer, with the advantage of greater portability.

The deflexions of a flexible diaphragm may also be used for measuring the varying pressures of steam, gas, and other elastic fluids; as in the *manometers* and pressure-gauges in common use.

127. *Illustrations*. The preceding articles may be illustrated and explained by the following, amongst other familiar illustrations.

Effects of the atmospheric pressure. The atmosphere, as we have seen, presses with a weight which is equal to 15 pounds on every square inch of surface, hence our bodies must sustain an enormous pressure, of which we are not sensible by reason of the equality of pressure which exists on all sides, and the consequent freedom of motion of our limbs without sensible resistance. But though we are not sensible of this pressure, we may easily become so. For let the equality of the pressure to which our bodies are subject

be destroyed by the removal of the air from any one portion of the body, then we are immediately made sensible of the enormous pressure which the rest sustains. For instance, let the air be exhausted by the air-pump from beneath a hand, which is placed over the open top of a cylindrical receiver; then the hand will be pressed down so firmly on the edges of the vessel that it will be impossible to raise it; and if the air be removed from one side of a piece of bladder tightly stretched over the open end of a receiver, the bladder will bend inwards more and more at every stroke of the pump, and finally burst. Let a tumbler or a cupping-glass have the temperature of the air increased within it by holding it over a spirit-lamp, then as the air within the glass becomes heated it expands, and a portion escapes into the surrounding atmosphere, and the elastic force of the air within the glass is in equilibrium with the pressure or weight of the atmospheric column at the mouth of the glass, when this expansion and its consequent motion ceases. Let the glass now be placed over any part of the body, then the pressure of the atmosphere on that part is replaced by the elastic force of the air in the cup, and no change is observable so long as the temperature of the air under the glass and its elastic force remain unchanged; but so soon as that temperature and elastic force of the air within the glass diminish, the flesh swells out into the glass by reason of the external pressure at the surface not being in equilibrium with the pressure below the surface of the skin.

The effects of this pressure as exhibited in pumps and siphons, in diminishing evaporation, and in preventing water from appearing to boil at a less temperature than 212 degrees, will be mentioned hereafter.

Inverted Vessels. A vessel full of liquid, as a tumbler of water, may be inverted, and the liquid will continue suspended whenever the height of the liquid column is not such

that the weight of any column of it is greater than the weight of the atmospheric column of the same section. But the equilibrium of the fluid will not in general subsist under these circumstances. For the equilibrium when it subsists being unstable, any disturbance causing the least displacement in any of the particles at the surface, will destroy the equilibrium. A piece of paper laid on the surface of the water, will insure the stability of the surface, and the vessel may then be inverted and the water will continue in equilibrium, the particles at the surface being insured from displacement by the rigidity of the paper.

But the most usual method of insuring this stability is to invert the liquid over a vessel containing another liquid, as in the barometer aud following examples.

Pneumatic Trough. The use of the pneumatic trough in chemical laboratories depends ou the pressure of the atmosphere. If any vessel be filled with a liquid and inverted, having its mouth plunged in a cistern of the same liquid, the liquid will remain suspended in the vessel, if the height of the vessel be not greater than the height of the column of the liquid which the atmospheric pressure will support.

When a vessel is so filled, if a tube be inserted under it communicating with a supply of gas, the gas passing through the liquid will rise in bubbles to the top of the vessel, and dislodge all the liquid from the vessel. Wheu the surface of the liquid in the vessel is in the same plane with the surfaee of the liquid over which it is inverted, the gas exerts the same pressure on the eontaining vessel as an equal quantity of the atmospheric air would do.

The Gasometer or *Gasholder* of the gas works may be mentioned here. It is made of various shapes and sizes, generally however cylindrical, and from about 30 to 80 feet in diameter, and 18 to 20 in height. It is closed at the top and open at the bottom, and inverted over a pit of water,

into which it sinks as it becomes empty, and out of which its edge cannot rise when full. Now the gas would not, under the pressure only of the atmosphere, flow with sufficient velocity through the mains and pipes to supply the lights; and the velocity with which it does flow may be regulated accurately by the weight of this gasholder, since the heavier it is the greater will be the density of the gas within it, and the greater the elastic force and velocity of the issuing gas.

127 *a*. *Atmospheric Railway.* The pressure of the atmosphere may be rendered available for the purpose of propulsion, in a manner which presents an instructive practical application of the laws of elastic fluids.

The weight of the atmosphere, or the elastic force of the air (Art. 126), produces a pressure which is the same on all sides of a body with which the air is in free contact; and if a piston be inserted in a horizontal tube or pipe open at both ends, the pressure of the air on each side of that piston is the same: if one end of the pipe be closed so as to cut off the connexion with the external air, other things remaining the same, the pressure on each side of the piston will still be the same, that is, the pressure due to the elastic force of the air in the closed portion of the tube will be in equilibrium with the pressure due to the weight of the atmospheric column in the open part of the tube. Now if any portion of the air be withdrawn from the closed portion of the tube by an air-pump, the density and consequently the elastic force of the air on the side of the piston from which the air is so withdrawn will be diminished, the equilibrium which subsisted between the pressures on the two sides of the piston will be destroyed, and the excess of the pressure on the side of the piston open to the air, over the pressure on the side of the piston from which air is so withdrawn, may be rendered available as a moving power.

The arrangements whereby this motive power may be obtained and rendered available for propulsion, as exhibited in the atmospheric railway, are as follow:

A cast iron pipe of about 9 inches internal diameter, and with a longitudinal slit on the upper side, is laid on the ground between the rails; this slit is covered by a longitudinal valve made of plates of iron and leather, or caoutchouc, fastened down to one side of the slit, about which side the plates are capable of moving as a hinge, the other edge of the valve being made to fit close to the other side of the slit, and rendered air-tight by some suitable composition. A piston is fitted to the pipe; to the stem or rod of the piston, but behind the piston, is fixed an arm, which, projecting through the slit of the pipe, (the valve being raised at its side,) is attached at its other end to the carriage which is to be propelled along.

At intervals of about two miles, (on the London and Croydon Railway,) stationary engines were erected to work air-pumps for the purpose of exhausting the air from the pipe. The pipe having been exhausted to the required degree, the excess of the pressure of the atmosphere on the one side of the piston over the elastic force of the air in the exhausted pipe on the other side of the piston, gave motion to the piston, which travelling along the pipe with a velocity (*cæteris paribus*) due to the degree of exhaustion, carried along the carriage and train attached to the piston-carriage*.

Instead of having the slit in the pipe closed by a valve lifted and opening at the side, it has been proposed to have the slit closed by a flexible diaphragm securely fastened down

* It would be foreign to our purpose to enter upon the various mechanical arrangements essential to the practical application of this principle, and for which we are indebted to Mr Clegg and Mr Samuda. It has been found too expensive and inconvenient in cases in which the locomotive engine can be employed, and is not now (A.D. 1855) in use.

at each side, and that the motion should be transferred from the piston in the interior, by causing the diaphragm to be raised by a wheel in the interior as the piston travels on, the raised portion of the diaphragm always pressing against a wheel attached to an arm projecting from the carriage to be moved forwards, and forcing it on. In this case suitable arrangements must be made for letting in the air behind the piston, so as to keep up the atmospheric pressure on the hinder side of the piston.

It has also been proposed to transfer the motion from the piston to the carriage, by means of two racks or toothed-plates, one in the inside and attached to the piston-rod, and the other on the outside of the tube and attached to the carriage; these racks are to work in two pinions, one at each end of a succession of spindles placed vertically in recesses by the side of the pipe, and turning in air-tight collars; provision being made for raising the spindles from their seats when the piston has passed by them, so as to let in the air behind the piston.

Whatever may be the mechanical details of the arrangements for working the atmospheric railway, the principle of obtaining motion by destroying the equality of pressure on the two sides of a piston in a pipe is the same in all. The piston cannot move until a certain degree of exhaustion has been obtained, and for this purpose it is necessary that the air-pumps should be at work some time before the train starts; in practice they are at work almost continuously, the pipe being divided into certain lengths by internal valves of a very ingenious construction; this preliminary or continuous working of the air-pumps affords an illustration of the accumulation of power, somewhat different from that presented by a large fly-wheel, or vessel of compressed air, the momentum and elastic force of which respectively are to be concentrated and expanded on some operation requiring great force, and to be performed instantaneously, or within a very

short time as compared with the period during which the power was being accumulated. The more perfect the exhaustion, the greater will be the available moving power; the theoretical limits of the degree of exhaustion will be similar to those already explained (Art. 99): in practice the degree of exhaustion varied from about 15 to 25 inches of mercury (Art. 100).

128. *Constitution of the Atmosphere.* Pure atmospheric air consists of 20 or 21 parts of oxygen, and 80 or 79 of nitrogen for every 100 measures. But the atmosphere is never absolutely pure, since there is always a certain quantity of carbonic acid gas, and of watery vapour, and probably of several other gases and vapours, diffused through it. Two views have been entertained of the nature of the union which exists among the several elastic fluids which constitute the atmosphere. The uniform nature of its composition, and the fact that the several ingredients continue mixed and do not separate and arrange themselves according to their specific gravities, seemed explicable only on the hypothesis of a chemical union.

But Dalton contends for a mere mechanical admixture of the different fluids in conformity with his law, that the particles of one fluid exert no action on the particles of another (Art. 94). Each ingredient of the atmosphere will then exert its own separate pressure in supporting the barometric column (Art. 95), and the whole column is supported by the different fluids in the following proportion.

The nitrogen atmosphere equals	21 . 2	inches,
... oxygen	7 . 8	
... aqueous vapour..............	. 6	
... carbonic acid.................	. 5	
	30 . 1	inches,

which is about the mean value of the atmospheric column*.

* Dalton, *Manchester Memoirs*, Vol. v.

129. *Limits of the Atmosphere.* If the atmosphere were supposed to be unlimited, it would pervade all space, and accumulating about the sun, moon, and planets, would form around each an atmosphere, the density of which would depend on the respective forces of attraction or gravity of the bodies. But it appears from astronomical phenomena, that there is no atmosphere about the heavenly bodies, and that our atmosphere therefore is confined to the earth.

Now the air decreases in density, and therefore in elastic force, as we ascend; and at some point or other the air may be so much rarefied, that its elastic force may be less than the gravitation of the particles, and at this point all tendency in the particles to separate farther from each other is counteracted by gravity; and if an equilibrium be established between these forces, the height of the atmosphere will be limited. The condition requisite that the limit may be possible, is, that the elasticity or repulsion of the particles of air should become less than the force of gravity on the particles. Now the loss of the elasticity will be owing partly to the distance which exists between the particles of air when highly rarefied, and partly to the extreme cold which prevails in the higher strata of the atmosphere; and these combined causes are quite adequate to bring about this equilibrium, and fix a limit to the height of the atmosphere*.

* See *Encyclopedia Metropolitana,* Art. *Meteorology.*

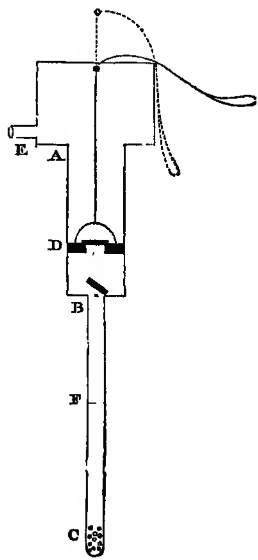
CHAPTER IX.

ON PUMPS, AND MACHINES FOR RAISING WATER.

Common Suction Pump.

130. When the air is withdrawn from a pipe, one end of which is immersed in water, the pressure of the air on the surface of the water, will force the water up into the pipe, and water may thus be raised to the height of 32 feet. This is effected by the common Suction Pump, which is represented in its simplest form in the accompying figure.

AB is a cylindrical barrel, and *BC* a pipe, termed the suction pipe, leading down to the water; at its upper end is a valve *B* opening upwards, and the lower part is closed and pierced with small holes to prevent masses of dirt from entering the pipe.



An air-tight piston *D* is moved up and down in the barrel by the pump-handle, which is a lever having the piston rod attached to its extremity. To explain the action of the pump, suppose the piston to be at the top of the barrel, then the pump-handle will be in the position represented by the dotted line. Both valves may be supposed to be shut. As the handle is raised, the piston descends, and the valve in it is opened by the action of the

air on its under surface; suppose the piston at the bottom of the barrel, then on depressing the handle the piston ascends, the valve in it is closed by the pressure of the air on its upper surface, and the air being removed from above the valve *B*, the valve is opened by the action of the air on its under surface, and continues open during the whole ascent of the piston, and the air in the suction pipe expands itself into the barrel.

The descent and ascent of the piston constitute one stroke of the pump; and it is evident that a quantity of air equal to the contents of the barrel is expelled from the barrel and suction pipe each stroke of the pump. But as the air is rarefied in the pipe *BC*, the water rises in it, for the elastic force being proportional to the density, the elastic force of the air in the barrel and suction pipe cannot sustain the pressure of the atmospheric column on the surface of the water, and the water will consequently rise in the pump as the air is withdrawn. Now when all the air is withdrawn, the water will be at the top of the barrel, and the next stroke of the pump will discharge from the pump through the spout at *E* a quantity of water equal to the contents of the barrel. This is the action of the common suction pump, and since its action depends on the water following the ascent of the piston to the top of the barrel, the weight of the sustained column cannot be greater than the weight of the atmospheric column, that is, *A* must not be more than 32 feet above the surface of the water in the well or reservoir from which it is to be raised.

If the top of the barrel be more than 32 feet above the surface of the water, but the bottom of the barrel be less, some water will be discharged each stroke; but if the bottom of the barrel be more than 32 feet, no water at all can be raised, and recourse must be had to other pumps, the simplest of which are the lifting and forcing pumps.

131. Prop. *The pressure on the piston during its ascent is the weight of a column of water whose base is equal to the area of the piston, and altitude equal to the height of the water in the pump above that in the reservoir.*

Let F (fig. Art. 130) be the surface of the water after any number of strokes of the pump: let r be the radius of the barrel, and h the height of the column of water whose weight equals the atmospheric pressure. Then πr^2 is the area of the piston, and $g\rho h \times \pi r^2$ is the pressure of the atmosphere on a portion of the surface of the fluid equal to the area of the piston (Cor. Art. 43); therefore pressure on under side of piston

$$= g\rho\,(h - FC)\,\pi r^2,$$

and the pressure of the atmosphere on the upper side of the piston is $g\rho h \times \pi r^2$, and the pressure on the piston during its ascent, that is, the force requisite to work the pump, is the difference of these.

Thus the force required

$$= g\rho h \times \pi r^2 - g\rho\,(h - FC)\,\pi r^2$$
$$= g\rho \times FC \times \pi r^2, \quad \text{or} \quad \propto FC.$$

Thus the labour increases at every instant, and is greatest when the pump discharges at every stroke. In practice the pump-handle gives a great mechanical advantage.

132. Prop. *To find the height of the water in the pump after any number of strokes.*

After n strokes let the water be at F, and let x_n be the distance BF, and let h_n be the height of the column of water which would be supported by the elasticity of the air in BF; and let x_{n+1} h_{n+1} be the values of these quantities after the $(n+1)^{\text{th}}$ stroke.

Let h be the height of the column of water whose pressure is equal to that of the atmosphere.

Let a be the length of the barrel and K its section, and b the length of the suction pipe to the surface of the water, and k its section.

Now the elasticity of the air above the surface at F, together with the weight of the column of water above the water in the reservoir, equals the pressure of the atmospheric column, and the portions of the surface on which these pressures are exerted are all equal;

$$\therefore\ h_n + (b - x_n) = h;$$
$$\therefore\ h_n - x_n = h - b.$$

Similarly $h_{n+1} - x_{n+1} = h - b$ ....(1).

Now the density of the air is as its elastic force; and by the $(n+1)^{\text{th}}$ stroke the air which occupied the space between B and F, that is, a length x_n of the suction pipe, is expanded over the barrel and a length x_{n+1} of the suction pipe;

$$\therefore\ h_{n+1}\{Ka + kx_{n+1}\} = kx_n h_n \text{ ....(2).}$$

Eliminating h_{n+1} between (1) and (2), we have

$$x_{n+1} = \tfrac{1}{2}\left\{b - h + \frac{Ka}{k}\right\} \pm \tfrac{1}{2}\sqrt{\left\{b - h + \frac{Ka}{k}\right\}^2 + 4h_n x_n}.$$

133. *Practical defects and limits to working.* In practice there is difficulty in making the piston and valves air-tight, the consequence of which is, that the piston may be worked up and down without raising the water; and when water is raised, which may be generally done by pouring a little water above the piston, which renders the joints and surfaces air-tight, the pump will "lose water" when no water is raised; that is, the water which is left in the barrel and suction pipe will gradually subside to the level of the water in the reservoir, and the pump will become empty.

If the piston does not descend to the bottom of the barrel, the air between it and the bottom may not have sufficient elastic force to open the valve, and if this be the

case, no more air can be expelled, and the water will not rise any higher.

Let the elasticity of the air in the barrel after the n^{th} stroke sustain a column of water whose height is h Now if the piston, instead of coming close to the bottom, descends only through the space a', and the valve is not opened, the air, whose elastic force or density $= h_n$, and whose bulk equals the contents of the barrel, is compressed into the space $(a - a') K$.

Let h'_n be its elastic force or density measured as before;

$$\therefore \; K(a - a') h'_n = Kah_n,$$

$$h'_n = \frac{a}{a - a'} h_n = \frac{h_n}{1 - \dfrac{a'}{a}}.$$

Now the pressure of the external air on the upper surface of the valve is measured by h, and the valve has not opened, therefore h is $> h'_n$. Hence the action of the pump will cease from this circumstance, unless h'_n, that is,

$$\frac{h^n}{1 - \dfrac{a}{a'}} \text{ is } > h, \quad \text{or } h_n > \left(1 - \frac{a}{a'}\right) h \ldots\ldots\ldots\ldots\ldots (1).$$

After the n^{th} stroke, we have (Art. 132) the condition

$$h_n - x_n = h - b;$$

$$\therefore \; x_n + h - b = h_n.$$

The valve will not open when

$$h_n = \left(1 - \frac{a'}{a}\right) h \text{ by (1)};$$

therefore when the action ceases,

$$x_n + h - b = \left(1 - \frac{a'}{a}\right) h,$$

$$x_n - b = -\frac{a'}{a} h; \quad \therefore \; x_n = b - \frac{a'}{a} h.$$

The action cannot go on unless

$$x_n + h - b > \left(1 - \frac{a'}{a}\right) h;$$

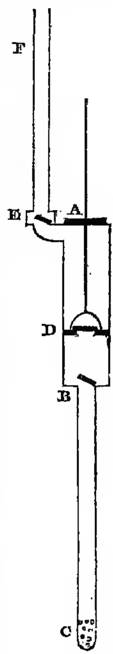
$$\text{that is, } x_n - b > \frac{a'}{a} h,$$

$$\text{or } b - x_n < \frac{a'}{a} h.$$

When the water is at the top of the suction pipe $x_n = 0$, and the action cannot proceed, that is, the water cannot be raised into the barrel of the pump, unless b is less than $\frac{a'}{a} h$.

The Lifting Pump.

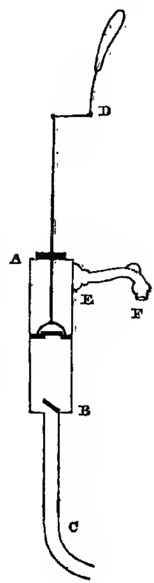
134. The water can only be raised by suction to about 32 feet, but it is frequently necessary to raise water for domestic purposes to the tops of very high houses, and in the draining of mines it has to be raised many hundred feet.



This may be effected by contriving that the water, which is raised by the atmospheric pressure to the top of the barrel, should by the ascent of the piston be *lifted* up into a pipe which reaches to the point to which the water is to be raised. For this purpose the barrel is closed and the piston rod works through an air-tight collar at A, and the spout E, through which the water would be discharged, if it were only requisite to raise the water to that point, has a valve opening upwards into the pipe EF, up which the water is raised. The action of the pump is evident from the accompanying figure, in which the piston is supposed to be ascending, and consequently lifting up a quantity of water equal to the content of the barrel into the

pipe EF, through the valve at E. When the piston begins to descend, the valve at E closes and prevents the return of the water into the barrel. Water may thus be raised to any height provided the barrel be strong enough to sustain the pressure of the superincumbent column of EF, and the force sufficient to work the common suction pump and sustain the weight of the superincumbent column EF, which must be raised before the valve at E can open. The top of the barrel must not be more than 32 feet above the level of the water to be raised, but the pipe EF may be any height; and the whole force requisite to work the pump is equal to the weight of the column whose base is the area of the piston, and altitude the height to which the water is to be raised (Art. 131).

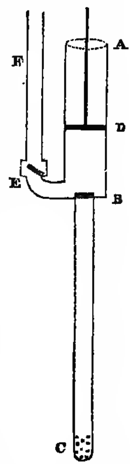
135. *Beer Machine.* The machine generally used in public houses for drawing beer from the cellar, and on board ship for drawing water from the casks, is an ingenious application of the lifting pump. The piston is worked in a small barrel about six inches long by a bent lever, moveable about a fulcrum D, so that when the handle is drawn down towards the body, the piston ascends from the bottom to the top of the barrel. The suction pipe or small tube BC has a universal joint at some point C, so that the end of the pipe, which is to communicate with the liquor to be raised, may be moved in any direction from one cask to another. Suppose the air to have been exhausted from the pump, and the barrel AB to be full of liquor, and the piston to be at the bottom of the barrel; then



on moving the handle, so as to bring it into the position represented in the figure, a quantity of the liquor flows out through the spout *EF*, and continues to flow during the ascent of the piston, that is, during the depression of the handle; but the instant the depression of the handle ceases, the liquor ceases to run, and the piston descending by its gravity, the handle resumes its former position.

The Forcing Pump.

136. The construction and action of the common forcing pump, by which water may be raised to a greater height than by the suction pump, will readily be seen by the accompanying figure. The piston *D* is solid, and the barrel *AB* open to the atmosphere. The water follows the ascent of the piston and fills the barrel; but the piston in its descent exerts a great pressure on the surface of the water beneath it in the barrel, and forces it up through the valve *E* into the pipe *EF*. The return of the water during the ascent of the piston is prevented by the valve at *E*. When water is to be raised to the same height, the force requisite to work the forcing pump is the same in amount as that which is required to work the lifting pump, but is applied in a different way. In the lifting pump the whole force applied to the piston rod is a tension in the direction of the rod's length. Hence a slim rod will do to work this pump, there being no downward pressure on the piston required. But in the forcing pump, the force required during the ascent of the piston is the same as in the common suction pump, but during its descent the force is a compression in the direction of the rod's length, hence the piston rod must be very stout, or it would bend under



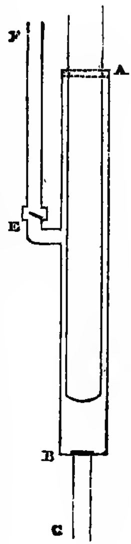
the pressure required to raise a high column of water; and this is frequently a great practical inconvenience. The kind of pump which is to be used must be determined by the peculiar circumstances of the case.

137. *Drainage of Mines.* In this instance, the machinery which is to work the pump is generally on the surface of the earth, that is, as high as the point to which the water is raised. The piston rods must therefore be very long; this being the case, the forcing pump, as just described, cannot conveniently be used, since a rod several hundred feet long capable of sustaining a pressure endways must be very stout, and the weight of such a rod would be inconvenient; and such a pump would be worked by the weight of the piston rod, and not in the manner just described. Forcing pumps then of this description are never used for raising water from great depths, unless it so happen that the moving power which is to work the pump is close to the pump. There are no similar objections to the lifting pump; for a very slender rod will bear a great tension applied lengthways, and the rod may be easily prevented bending from its own weight, by placing two or three collars through which it may work and be kept by them in a vertical position.

Plunger Pole Pump.

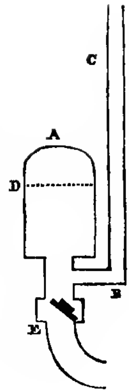
138. Great difficulty is experienced in practice in making the pistons move water-tight in the barrels, especially when dirty water is to be pumped up, as in the drainage of mines; and also when great pressure is to be applied to the water, as in the forcing pumps of the Hydro-Mechanical or Bramah Press. Hence forcing pumps with pistons accurately fitting the barrel are almost entirely superseded in cases of this kind by the plunger pole pump. The piston of this pump is a solid plunger of the same size all the way up, but not quite so large as the barrel, and it works through a water-

tight collar or stuffing box at *A*. The action of the pump will be readily comprehended from the figure. The withdrawing of the piston causes a partial vacuum in the barrel, and the water rises through the valve at the top of the suction pipe *BC*. When the plunger descends the water is forced through the valve at *E* and raised up the pipe *EF*. There is but little difficulty in keeping the pump water-tight by means of the stuffing box at *A*, and as the material wears away a fresh quantity may be readily supplied. This is a most important practical contrivance, and dispenses almost entirely with the necessity of boring large cylinders accurately true, which is a task of no ordinary difficulty: and when performed, the contact of the edge of the piston with the metal surface of the barrel would soon wear away the piston and the barrel, so that the pump would not act, and this could not be repaired without removing the piston. But in the plunger pump the only wear is on the surface of the plunger and the stuffing box, which may be repaired without any difficulty.



The Air Vessel, or Air Spring.

139. The stream discharged by all the pumps which have as yet been described is intermittent, that is, the water sometimes ceases to run. Thus in the common suction pump the water will not flow during the descent of the piston, unless the quantity raised by each stroke of the pump into the pump-head takes the whole time of a stroke for its discharge at the spout. And in the lifting and forcing pumps the stream must cease to flow during the descent of the piston of the former and the ascent of the piston of the



latter. But the stream may be made continuous by the addition of an air vessel. The water after being raised through the valve at E (see also figs. Arts. 134, 136, 138) which is called the Forcing Valve, is received in any close vessel full of air, as represented at A. The water occupies the lower part of the vessel, and as more water is forced in the air becomes more and more compressed. The pipe BC, through which the water is to be raised, communicates with the lower part of the vessel, so that its orifice is always below the surface of the water.

Now the elastic force of the air is inversely as the space it occupies (Art. 89); hence when the air becomes compressed into a very small space, the action exerted on the surface of the water in the air vessel by the elastic force of the compressed air will raise up and sustain a very high column of water in the discharge pipe BC, and the height to which it is raised will be proportional to this elastic force, so that if the vessel and pump be strong enough, there is no theoretical limit of the height to which the water may be raised. If the air vessel be large enough, and the discharge pipe be somewhat smaller than the supply pipe, the quantity of water in the air vessel will not be much increased by each stroke of the pump, and the elasticity of the air being a force constantly acting, the stream will run continually and with a velocity nearly uniform.

The term *Air Spring* has most appropriately been applied to the apparatus just described, since it obviates the same practical evils as springs of carriages, and is as necessary in large pumps as springs are for the preservation of the roads and carriages.

The degree of condensation of the air in the air vessel may be measured by the siphon gauge of the condenser (Art. 105).

Artificial fountains may be constructed on this principle, and one air vessel may supply a great number of jets.

140. *Water Works.* The invention of the air vessel in a great measure supersedes the necessity of the former expensive process of raising all the water into a reservoir which is above the level of all the places to which the water is supplied (Art. 35). A vast power is thus expended unnecessarily, since it is very probable that more than half the water is wanted at levels greatly inferior to the level of the reservoir. A plan which is frequently resorted to, consists in having a very large air vessel, and two forcing pumps, one much larger than the other, and worked *in succession* by a steam-engine. The larger pump is worked first to supply all the lower levels with water, and when this is done, the supply being cut off by means of the street-cocks, the water is diverted into the pipes which are to supply the higher levels. Here now a much higher column is to be raised, and consequently the work to be done by the steam-engine is much greater. The smaller pump is now set to work, and the column of fluid which is to be raised having a piston of less diameter to act on, the steam-engine can work as easily as before, although producing a much greater condensation of the air in the air vessel, in consequence of which the water may be raised to a much greater height.

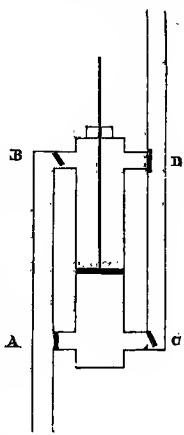
141. *Defects of the Air Vessel.* Every portion of water in its natural state has a certain quantity of air combined with it, and this quantity may be increased by subjecting air in contact with water to pressure. Hence in the air vessel, the air being constantly subject to very great pressure, and in contact with water, is absorbed by the water

and passes away with it, so that the air vessel becomes full of water and the stream ceases to be continuous. To remedy this, a very small hole is made in some part of the suction pipe, which is always exposed to the air, in order that a quantity of air may be drawn up with the water at every stroke of the pump, and delivered with the water into the air vessel. Thus the supply of air is kept up; and for small pumps an ordinary pinhole is sufficient, but in large pumps a large hole is made and fitted with a stop-cock, which may be opened when necessary.

De La Hire's Double-acting Pump.

142. The lifting pump and forcing pump are open to a common objection, namely, the loss of power which arises from the necessary friction between the piston and barrel when they are doing no work. Now the full effect of two pumps, with the friction and expence of one working barrel only, may be obtained by an ingenious combination of the lifting and forcing pump.

The construction and working of the double-acting pump will be evident from the accompanying figure.

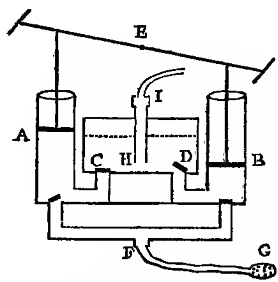


The piston is supposed to be descending, and consequently the forcing valve at *C*, communicating with the discharge pipe, is open, and the valve at *A*, communicating with the supply or suction pipe, is shut. This may be considered as the part of the common forcing pump which enters into the construction. But the valve at *B*, communicating with the supply or suction pipe, is open, and the valve at *D*, communicating with the discharge pipe, is shut. This is the part of the lifting pump which enters into the combination.

During the ascent of the piston the valve at *A* and *D* will be open, and at *B* and *C* shut. Then, as the piston descends, water runs into the barrel at *B*, and the water below the piston is forced through at *C* up the pipe *CD*, and as the piston ascends, water runs in at *A* and the water above the piston is lifted out at *D*, so that two barrels of water are raised each stroke of the pump, and the stream would be continuous, or very nearly so, without the use of an air vessel. In practice, however, there is an awkward jar at each change of the action, which is almost entirely removed by the use of the air vessel. This is a very convenient pump, and being used extensively for supplying air to furnaces, is sometimes called a *Double-acting Air Pump.*

The Fire Engine.

143. The fire engine consists of two powerful forcing pumps, placed opposite each other, with one air vessel between them. The pumps are worked by a common lever, having its fulcrum at *E*, midway between the pumps; so that when the piston of one is depressed the other is elevated, and thus one or other of the pumps is always forcing water into the air vessel. The water is supplied to the pumps by a common suction pipe, *FG*, of leather, having its extremity *G* placed in the supply of water. The elastic force of the air in the air vessel acting on the surface of the water, which is represented in the figure by the dotted line, forces it up the pipe *HI*, into a long leather pipe or *hose* which is attached at *I*, and may be carried in any direction. At the other end of this hose is a brass spouting pipe or



adjutage, which serves to direct the water forced through the hose accurately to any point, and being contracted towards the orifice, the water may be thrown to a very great height, and with a force sufficient to break any windows through which it may be desirable to direct the water.

The action of the pumps will readily be seen from the figure. The piston in *A* is ascending, and in *B* descending, consequently the forcing valve *C* is shut and *D* is open. The fire engine is generally made long and narrow in its external case, which hides the pumps, and the principal use of this length is to support a long framework for the lever, that the pumps may be worked by the united strength of several men, standing on each side of the engine. The long body of the engine serves likewise as a cistern, into which water may be poured when there is no convenient supply, into which the end of the suction pipe may be placed. The pumps may communicate with this cistern, either by a plug for the purpose, or by turning the end of the flexible suction pipe into the cistern.

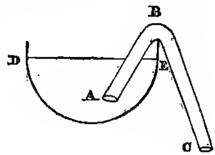
The height to which the water will be thrown depends on the degree of condensation in the air vessel, and on the elevation of the extremity of the pipe above the level of the water in the air vessel. For the elastic force of the air in the air vessel has to support a column of water, the height of which is equal to the difference of the levels of the end of the spout and of the water in the air vessel, and until the elastic force of the air exceeds the weight of this column, no water can spout out. The height to which it will spout may be increased by increasing the condensation in the air vessel, which is done by placing the hand over the orifice of the spout, and so stopping the discharge of the water for one or more strokes; thus more water is forced into the air vessel, and the air becoming more condensed, exerts a greater elastic force; and should all the air

be conveyed away from the air vessel, a sufficient quantity may be forced in, by working the engine for a stroke or two, having removed the extremity *G* of the supply pipe out of the water.

The Siphon.

144. If a tube bent at *B*, having one leg shorter than the other be full of water, and the shorter leg be placed in a vessel of water, the water will run out through the longer leg *BC*.

The siphon is generally filled by suction; the air is withdrawn from the tube at the end *C*, the end *A* being inserted in the water, and the pressure of the atmosphere on the surface of the water raises it in the leg *AB*, and the water reaching *B* runs down and fills the longer leg *BC*. The siphon being thus entirely full of water, may be considered as consisting of two separate columns, occupying the two legs of the instrument, and having a tendency to separate at the point *B*; the short column having a tendency to run back into the vessel, and the long column to run out at *C*. But then there would be a vacuum produced at *B* by the separation of these columns, which, when the tubes are less than 32 feet is impossible, but the columns are sustained by the atmospheric pressure at the orifices of the tubes; at the orifice *A*, by the pressure transmitted from the surface of the water in the vessel; and at the orifice *C*, by the pressure exerted directly. But the pressures of these columns are equal, for the elevation of the surface of the fluid above the orifice is too small to cause any appreciable variation in this weight. Hence the columns of water which these pressures will sustain are equal; but here they are not, for one leg is longer than the other, motion therefore



will ensue, and the longer column BC will move down and be discharged at the orifice at C, and the atmospheric pressure will keep the leg AB constantly full.

Thus the water circulates through the tube, and the only circumstances to be observed are, that the column of fluid between B and the surface of the fluid in the vessel must be less in height than the column in the other leg, and less than 32 feet, or in the former case, the fluid will run back again into the vessel, and in the latter, it will separate at B, since each column is heavier than the atmospheric column.

Thus the motion of the fluid is similar to the motion of a chain hanging over a point. If the two parts of the chain be equal, the fluid remains at rest; and if one end be longer than the other, it moves in the direction of the longer end. Fresh links, so to speak, are added continuously to the fluid chain, by the atmospheric pressure on the surface of the fluid, so that the chain being continuous, the motion is continuous also, and does not cease till one portion of the chain becomes equal to, or less than the other. That this is really the case, will be seen at once on inspecting the action of a siphon under the receiver of an air-pump. The water runs more and more languidly every stroke of the pump, and finally stops, since no new links are added to the chain, but on the readmission of the air it commences running again, and when all the air is readmitted runs just as before.

145. *Wirtemberg Siphon.* When the columns are nearly equal, and consequently the circulation languid, the air steals up the discharging leg, and collecting at the top of the bend separates the columns, and the action is suspended. This may be avoided by making both legs equal, and turning them up for about an inch at the extremities. Then the siphon always continues full, and when one end

is dipped in a fluid the other begins to run, and continues till the surface of the fluid coincides with the surface of the orifices.

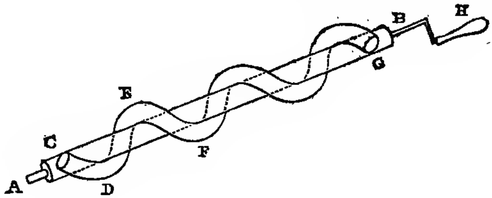
Siphons are used for drawing off liquids from the upper parts of vessels containing them, without tapping or making any orifice below. They are highly useful in many chemical processes for drawing off a liquid without disturbing the sediment. Large siphons are generally furnished with a stop-cock near their lower extremity; and a small tube to which the mouth may be applied communicates with the leg just below the stop-cock, so that the stop-cock being open, and the hand applied to the orifice of the leg to stop the communication with the external air, all the air may be exhausted by suction, and the legs filled with the liquor which is to be drawn off. The siphon being now full, the liquor will run when the stop-cock is opened.

146. *Intermittent Springs.* The action of *reciprocating or intermittent springs*, that is, of springs which flow and cease, may be readily explained on the principle of the action of the siphon. Water is collected from various sources in a large cavern in the interior of the earth, and the only outlet for the water is by means of a bent channel, the bend of which, like the bend of the siphon ABC (fig. Art. 144), is raised some feet above the usual level of the water in the cavern. Now when the cavern becomes very full, so that the water stands at a higher level than this bend, the water fills this bend and runs out and continues to run, as in the siphon, when the level of the water is much below the bend.

The Screw of Archimedes.

147. A tube is wound in a spiral manner, as represented in the accompanying figure, about a cylinder whose

axis is AB, which is placed with its end B much more elevated than the end A, and capable of being turned



about by a winch at H. The circumstance to be attended to in the degree of inclination of the axis and spiral tube is, that the point D must be just below the orifice C, and the point F just below E, and so on throughout the tube; that is, whatever be the position of the spiral, the level of any bend on the upper side must be just above the level of the next bend on the under side, reckoning along the axis from the lower to the upper end.

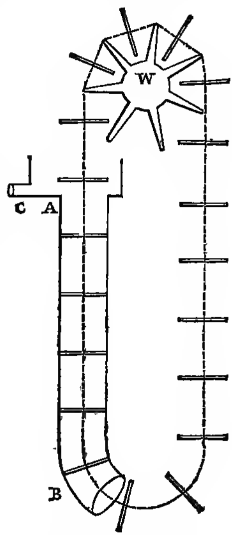
Let a bullet be put into the tube at the orifice C, in the position of the screw represented in the figure, it will immediately run down to D. Let the screw be turned half round, then D will be just above the level of E, and the bullet will have run to E; let it be turned once more half round; these make together one complete turn, and the screw will again be in the position represented by the figure, that is, E will be just above F, consequently the bullet will have run down to F. Similarly, the screw being turned once and a half more round the bullet will drop out at G.

Now a fluid mass obeys the same laws of motion as a solid (Art. 10), hence if the end A of the screw be immersed in water, so as to cover the orifice C, and the screw be worked, after two turns and a half, in a screw such as is represented in the figure, the water will begin to run out at G, and the stream will be continuous during the motion

of the cylinder. The screw-pump is a very useful machine for raising large quantities of water to a small height; and has this advantage over common pumps, that it will raise water mixed with gravel, dirt, or other impurity, without being deranged in any manner.

The Chain-Pump.

148. An endless chain composed of links, united together so as to bend readily, is placed on the spokes of a wheel *W*. On this chain are fixed at equal distances from each other a number of pistons all of which fit accurately the barrel *AB*. As the wheel *W* is driven round, the spokes catch successive points of the chain, and the pistons pass up the barrel *AB*. Now if the lower end of the barrel be immersed in water to a depth at least equal to the distance between two of the pistons, as these pistons successively enter the barrel they will carry up the water with them, and discharge it at the spout at *C*. If the wheel revolves with great rapidity a considerable quantity of water will be raised, even if the pistons do not fit very accurately. The great advantage of this pump is, that it can work in the foulest water, and will bring up stones or any materials which enter the bottom of the barrel.



CHAPTER X.

ON THE MOTION AND RESISTANCE OF FLUIDS.

149. Fluids are subject to the same law of gravity as solid bodies, and a mass of fluid descending vertically has its motion accelerated in the same manner as a solid mass; and the momentum generated is the product of its mass and velocity. If a column of water move through a vertical or inclined pipe, it acquires a velocity, which from the friction of the pipe will soon become uniform, and the motion which it possesses, or the momentum generated, is the mass of the column multiplied into this uniform velocity. Now force is always necessary for the destruction of motion, and the shorter the time through which it acts the greater is the effect produced*. Thus a small hammer with a hard face is much more effective in driving a nail, than a mallet of twenty-times its weight, and moved with the same velocity. For in consequence of the hardness of the face the motion is destroyed instantly, and consequently the effect exerted on the nail is very great. The sudden destruction of motion in a fluid mass is attended with effects precisely analogous. When the motion of a fluid mass is suddenly stopped, the surface which stops it must sustain a very great force. The great shock which the gates of a lock experience when the motion of the stream, however slow that motion, is stopped by the gates closing completely, is a familiar illustration of this fact. Now when the discharge of a quantity of water from a pipe is suddenly

* The equation which expresses this is, $v = ft$, or $f = \frac{v}{t}$, where f is the accelerating force which generates the velocity v in a time t. Now the same equation holds for a retarding force. Hence it is evident, that when t is very small during which the velocity v is destroyed, the effect must be very great.

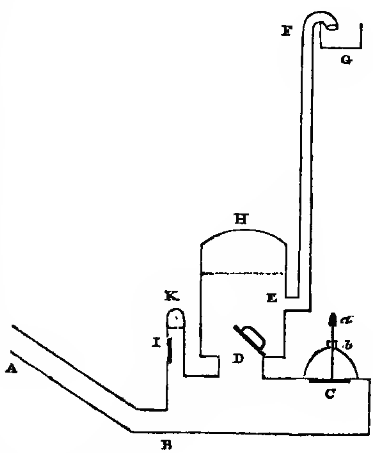
stopped, the action sustained by the opposing surface is counterbalanced by a reaction, which is transmitted through the whole mass (Art. 23), and impressed on every portion of the containing vessel.

The action thus sustained by the sides of the pipe is frequently sufficient to burst it, and this may take place not only close to the stoppage, but at any distance, wherever the pipe happens to be weak. If a large main, for instance, running with a great velocity were suddenly turned off, every small pipe which communicated with the main above the point at which the stoppage took place, would probably be burst.

The action thus exerted on the instantaneous destruction of the momentum generated in a column of water in motion, has been most ingeniously applied to raising water to a great height. The combination of vessels by which this is effected, is termed the Hydraulic Ram.

The Hydraulic Ram.

150. *AB* is the pipe leading from the reservoir or head of water. *C* the valve through which the water runs out till it has acquired nearly a uniform velocity; the valve can drop down through the space *ab*, the knob *a* preventing its falling quite out.



K and *H* are air vessels, and *EF* the pipe up which the water is to be raised to the vessel at *G*.

Suppose the valve C open, and the valve D shut, and that the water is moving down AB and escaping through C, now when the water moving down AB has acquired a certain degree of velocity, the valve C is shut by it; the motion of the water is then stopped in an instant; a great action is thereby produced on every part of the containing vessels, and the valve at D is forced open, so that the water rushes into the air vessel H, and the air being condensed sufficiently, the water is raised in a continuous stream to any height G. The valve at C is now closed, and before the action of the ram can proceed, or another pulsation can be made, it must be opened again.

This is effected by the weight of the valve, with the assistance of the air vessel at K.

The action consequent on the close of C condenses the air in K into a very small compass, when then from the opening of D and the slight recoil of the water up the pipe BA, the action on the air in this vessel is relieved, its elastic force drives the water still farther back up the pipe BA; the consequence of which is, that at the instant the water begins to descend the pipe AB, the pressure on the under surface of C is almost entirely removed, and the valve C drops by its weight. But the air in K is speedily absorbed, and a small quantity is supplied every pulsation of the ram by a valve at I opening inwards. For the air in K being suddenly condensed and as suddenly expanding again, expands or recoils, so to speak, too far, so that its elastic force becomes less than the elastic force of the external air; the valve at I opens, therefore, at that instant and admits a small portion of fresh air.

When a given quantity of water is to be raised to a great height by the ram, the momentum generated must bear a certain proportion to that height. The momentum generated will depend on the length of the column AB, and on

the height from which it descends. The quantity of water raised by this machine is about 66 per cent. of the quantity used, and the number of pulsations vary from 50 to 70 a minute.

Water Wheels.

151. *Undershot wheel.* A common mode of deriving power from the motion of water, is by submitting a number of flat boards placed on the rim of a large wheel to the action of a running stream. The impulse of the water, as it strikes one board, drives round the wheel, and another board is immediately immersed in the stream, and this board receiving a similar impulse, the wheel is carried round as before. The wheel is thus caused to revolve in the direction of the stream with a velocity which depends on the velocity of the stream, and on the number, form, and position, of the float boards.

Overshot wheel. Another mode of obtaining power is by an overshot water-wheel, in which the weight of the water acts to drive round the wheel. On the rim of a large wheel are constructed a number of cavities called buckets, and as the stream fills the top one, it descends causing the wheel to revolve, and an empty one succeeds it which is likewise filled, and the wheel is kept revolving by the buckets going down full on one side and coming up inverted, and therefore empty, on the other.

Much ingenuity has been displayed in determining the best forms for buckets, and the best size and velocity for wheels. But these details cannot be given here; they will be found in Treatises on Mill Work.

Breast wheel. A third species of wheel is the breast wheel, which is driven both by weight and impulse. The boards are flat, and the water is delivered nearly on a level with the axis of the wheel, the mill-stream below that point being made circular to suit the wheel, so that the edges of the

boards are nearly in contact with it as the wheel is driven round.

151 *a.* *Water-Pressure Engine.* In machines described in the last Article under the denomination of water-wheels, the water acts by impulse or by weight, or by weight and impulse combined; to these may be added a machine which acts principally by pressure, and which has accordingly been designated as the water-pressure engine. This engine in the mechanical arrangements employed closely resembles the single-acting steam-engine (Art. 187); the pressure of a head of water being the moving power instead of steam. In some cases the engines are made double-acting, in which cases the arrangements are the same as the double-acting steam-engine (Art. 188); the water may be conducted from any convenient reservoir by a small pipe to fill the cylinder in which the piston works, and when the piston has completed its stroke by means of the pressure of the head of water, the contents of that cylinder are run off, and the piston makes another stroke, either by the action of counterbalance weights (as in the atmospheric engine, Art. 185), or by the pressure of the head of water on the other side of the piston. The water-pressure engine is a very valuable machine in mining and mountainous districts, where fuel is scarce; inasmuch as a head of considerable height may frequently be obtained at a very small expense, a small pipe only being required for the purpose.

152. *Reaction Machines.* There are several instances in which motion is given to machinery by the lateral pressure of the fluid not being counterbalanced, and machines thus moved are called reaction machines. When a fluid is contained in a vessel, every part of the vessel has a certain pressure exerted upon it, which is counteracted by the reaction of the side. If a portion of the side be removed, so that this reaction cannot be called into play, the reaction of

the opposite side will not be counterbalanced, and motion may ensue. A tall vessel full of water standing on a horizontal plane has no tendency to move, but if a hole be pierced in the side at the upper part of the vessel, the vessel will under certain circumstances be tilted over or overturned on the side opposite to the hole (Art. 52).

This principle is applied in Barker's Mill, which consists of two tubes fixed as horizontal arms into the bottom of a vertical tube, supported in a collar at the top and on a point at the bottom, and capable of moving about a vertical axis; near the extremities of each of the horizontal arms, but on opposite sides of each arm, is an orifice through which the water supplied at the top of the vertical tube may flow; if these orifices were closed, the machine would be at rest, but being open and on opposite sides of each arm, the pressure of the water on the sides of the horizontal arms is not balanced, but they move in opposite directions, thus causing the whole machine to turn about the vertical axis. The above however is not so economical a mode of applying a head of water to produce motion as the overshot wheel.

An improvement upon the above may be made, by curving the horizontal arms in such a manner, that the water as it passes from the centre may move in a straight line by reason of the arms, owing to their curvature, receding as the water advances.

An elastic fluid, as steam, or hot air issuing out from orifices at the end of an arm which can revolve about an axis in a direction opposite to the issuing stream, may be applied to produce motion.

It has been proposed to move vessels through the water by the efflux of water and highly elastic gases at the stern.

152 *a.* *Turbines.* The most improved form of any machine of this class, appears to be that known by the name of the *turbine;* which is a horizontal wheel bounded at

its upper and under sides by plain horizontal surfaces, and accurately fitted to a fixed vertical cylinder down which the water passes, and acts on the curved side of channels open at both ends, extending from the external circumference of the vertical cylinder to the outer circumference of the horizontal wheel.

The curved sides of these channels are arranged so as to take advantage of the centrifugal force of the water as well as its pressure and velocity.

The turbine can work under water, and with any available fall or head; advantages peculiar to this machine*.

152 *b*. *Resistance of Fluids.* The reaction of fluids is further illustrated by the resistance opposed to the motion of a body; if any body be moved through a fluid, the resistance will be proportional to the area or surface impinged upon, or in contact with the fluid; and for equal areas moved at different velocities, the resistance will be nearly as the square of the velocity. The determination of the precise law of the resistance of fluids to bodies, and of the forms of least resistance, are questions of considerable difficulty, and to which the attention of practical men has been much directed †; the resistance of the air to the passage of railway trains at high velocities, has given rise to many important experiments, the general result of which appears to establish the law, that the resistance for equal areas is nearly as the square of the velocity. In the case of bodies moving rapidly through the air, an atmosphere of the fluid as it were, adheres to the body and moves with it, thus increasing practically the resisting surface of the body.

* See D'Aubisson's *Hydraulics*, and papers by Professors Rankin and Thomson in *Reports of British Association*, for further information on this subject.

† See papers by John Scott Russell on this subject in the *Reports of the British Association*, *Transactions of the Royal Society of Edinburgh*, and Beaufoy's *Experiments*.

Paddle-wheels. The use of paddle-wheels for the purposes of propulsion in steam-vessels, is an instance in which the momentary or instantaneous resistance of a fluid to the motion of a plane surface, is rendered available for impelling the vessel forward through the water.

Screw Propellers. The resistance to a plane moved obliquely in water, or of water impinging obliquely on a surface, is well illustrated by the screw propellers which have recently come into use for the propulsion of vessels. A screw of half a turn, or of a portion of a turn about an axis being made to revolve rapidly about its axis, so that the surface of the blade meets the water obliquely, will experience such a resistance in the direction of its axis, as to occasion motion in a ship or other body to which it is attached. The resistance in the case of impact between a surface and a fluid, may be resolved in directions parallel and perpendicular to the surface, according to the laws of mechanical science.

Sails of a Windmill. A further illustration of the reaction of a fluid in motion, is afforded by the sails of a windmill, which are driven round in a plane at right angles to the direction of the wind, by the action of the air upon the surfaces opposed obliquely to its action. It will be evident that in any of the preceding cases, the result will be the same, whether we suppose the resisting surface to be at rest, and the fluid to impinge upon it with a certain velocity, or the fluid to be at rest, and the surface to impinge thereon, or both to be in motion, provided their relative velocities be the same as before.

153. *Fluids flowing through orifices.* The determination of the motion of fluids is a question of the greatest theoretical difficulty, and can only be solved in particular cases and on particular hypotheses.

When the motion is *steady*, that is, when the velocity

is always the same for the same point in space, we obtain the following result*;

$$v = \sqrt{\frac{2gz}{1 - \frac{k^2}{K^2}}},$$

where v is the velocity, g the force of gravity, z the depth of the orifice below the surface of the water, k the area of the orifice, and K the area of the surface of the water retained at a constant height.

When the orifice is exceedingly small compared with the surface of the fluid in the vessel, the square of the ratio of their areas may be omitted; consequently,

$$v = \sqrt{2gz},$$

or the velocity is that due to the height from which the particles have descended; that is, it is the same as the velocity acquired by a heavy body falling freely under the action of gravity through a space equal to the depth of the orifice below the surface of the fluid.

Now when a fluid issues through an orifice the stream is convergent for a small distance from the vessel; it then acquires a permanent form, neither converging nor diverging; and this portion of the stream is called the *vena contracta*. It is the area of the section of this contracted vein, and not the area of the actual orifice, which being substituted for k in the above equation, will give the true discharge†.

The area of the section of the *vena contracta* may be taken as equal to $\frac{5}{8}$ths of the area of the actual orifice‡.

For further details connected with this subject, the student is referred to Venturi's *Experiments on the Motion of Fluids*, and generally to works on practical hydraulics.

* *Theory of Fluids*, Arts. 116—121.

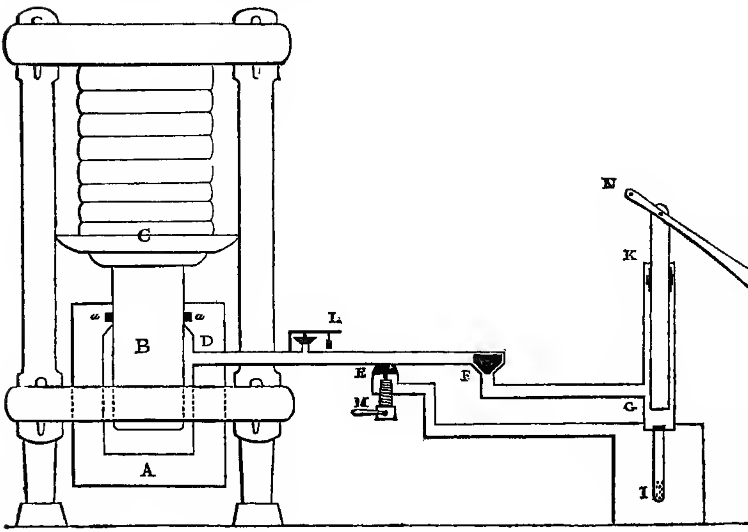
† *Theory of Fluids*, Art. 118.

‡ Rennie's *Report to British Association*, 1834.

CHAPTER XI.

ON THE HYDRO-MECHANICAL OR BRAMAH PRESS.

154. *A* is a strong cast iron cylinder, *B* a piston moving in it and carrying the table *C*.



DEFG the conduit pipe by which the water is conveyed from the injecting pump to the working cylinder.

KGI the injecting pump with the water cistern below.

N the fulcrum of the lever by which the pump is worked, *F* the forcing valve, *L* the safety valve, *E* the

discharge valve, which is opened and shut by the screw *M*, thereby opening a communication between the working cylinder and water-cistern.

The cylinder *A* is firmly fixed in a vertical position in a frame-work of iron of sufficient strength to sustain the enormous pressures which are exerted on substances placed between the table *C* and the top beam of the frame. The piston *B* is raised up by the pressure of water on its bottom, and is rendered water-tight by a leather collar represented at *a, a*, which will be explained more fully in a following article. The piston fits the cylinder just on each side of this leather very accurately, by which a steady and vertical ascent is insured; but the diameter of the cylinder, excepting just at the top, is somewhat larger than the diameter of the piston; and the orifice *D* at which the water enters may be situated any where, as is most convenient, for the pressure which is exerted on any part of the fluid will be transmitted in every direction and exerted on every equal portion of the piston. The pressure on the bottom is the only effective part, since the pressures on the sides counteract each other. The injecting pump is a plunger pump (Art. 138), and the plunger piston works through leathers of a peculiar construction. Since on every descent of the piston a quantity of water is forced through the valve at *F*, which closing the instant the piston has completed its descent prevents the return of the water, the water in the working cylinder *A* must either be compressed or must occupy a larger space. But since for all practical purposes water is incompressible, the volume of the water must increase every stroke, and since the vessel is too strong to yield, the piston *B* must rise. Thus the piston must continue to rise so long as any additional quantity of water is forced in. When the required effect of the press has been exerted, the pressure is immediately relieved by opening the discharge valve *E*, when

the water returns to the cistern and the piston descends by its gravity.

155. Prop. *The force which is applied to the injecting pump being given, to determine the action produced on the base of the large piston.*

Let K be the section of the working piston, and k the section of the plunger of the injecting pump, and P the force applied at the pump-handle.

Let L be the distance from the fulcrum, or the length of the lever at which the force P is applied, and l the distance of the connexion of the plunger with the handle from the same point.

Then the force applied to the plunger by the given force P, acting at such a lever, is $Q = P \cdot \frac{L}{l}$ (*El. Mechanics*). Now the pressure exerted is proportional to the area pressed, and the pressure exerted by the plunger at any instant on the water in the injecting pump is transmitted to the base of the large piston;

$\therefore$ the action on the base of the large piston

: pressure exerted by the plunger :: $K : k$.

But the pressure exerted by the plunger $= P \cdot \frac{L}{l}$;

$\therefore$ the action on the base of the large piston $= P \cdot \frac{L}{l} \cdot \frac{K}{k}$.

This then is the pressure produced on any substances placed between the table and the top beam of the framework, or it is the weight raised, if the press be so applied; calling it W, we have

$$W = P \cdot \frac{L}{l} \cdot \frac{K}{k}.$$

Let R and r be the radii of the sections K and k, then $\frac{K}{k} = \frac{R^2}{r^2}$;

$$\therefore W = P . \frac{L}{l} . \left(\frac{R}{r}\right)^2.$$

From this equation it is evident, that the effect produced may be increased by the increase of any of the quantities P, $\frac{L}{l}$, or $\frac{R}{r}$, which compose the product.

156. Let us examine the action in some particular instances, such as generally occur in practice.

Ex. 1. A man working at the injecting pump exerts by his hand a pressure nearly equal to his weight. Let $P =$ 112 lbs., or 1 cwt.

Let the pump-handle, or lever at which he works, be 3 feet long, and let the plunger be connected with it at a distance of 3 inches from the fulcrum. Then

$$\frac{L}{l} = \frac{36}{3} = 12.$$

The diameter of the large piston is very frequently 6, 9, or 12 inches, and of the plunger 1 inch. Take 10 inches as the diameter of the piston and 1 inch for that of the plunger.

Then $\frac{R}{r} = 10$.

Substituting these values in the equation,

$$W = 112 \text{ lbs.} \times 12 \times (10)^2 = 134400 \text{ lbs.}$$
$$\text{or} = 12 \times 100 \text{ cwt.} = 60 \text{ ton.}$$

Thus by one man working at the pump a pressure of 60 ton is produced, and if two men work a pressure of 120 ton will be produced.

Ex. 2. Small presses are very often made, where only small pressures are required, or as models merely to exhibit the power of the press.

Let the diameter of the piston be 2 inches, and of the plunger one-fourth of an inch. Then $\frac{R}{r} = 8$.

Let $\frac{L}{l} = \frac{12}{2} = 6$, and $P = 20$ lbs., which is easily produced with the hand without any effort.

Then $W = 20$ lbs. $\times 6 \times 64 = 7680$ lbs. $= 68$ cwt., or a mere child could raise a platform on which more than 50 grown people were placed.

Ex. 3. To shew the enormous power which may be exerted by the press, let the piston be 20 inches in diameter, and the pump one-fourth of an inch. A press of this description has been made.

Then $\frac{R}{r} = 80$. Let $\frac{L}{l} = 12$, and $P = 112$ lbs. as before.

$$\text{Then } W = 1 \text{ cwt.} \times 12 \times (80)^2 = 12 \times 6400 \text{ cwt.}$$
$$= 3840 \text{ ton.}$$

And two men working at such press would produce a pressure or raise a weight of 7680 ton.

157. From the preceding examples, it is evident that the great increase in power exerted arises from the increase of the ratio $\frac{R}{r}$, since the power is multiplied by the square of this quantity, instead of only by the first power. And the press may be said to have no limit to the pressure which may be exerted, except the strength of materials; for the ratio of $\frac{R}{r}$ may be increased almost indefinitely. But practical considerations come in, and an important one to be remarked is, that when the ratio $\frac{R}{r}$ is large, the quantity of water injected every stroke of the pump is so small, that the piston ascends too slowly for practical convenience; and a nine-inch piston, with a one-inch pump, is found a very convenient combination for general purposes.

It is desirable to save time in every operation; hence, since the work during the first ascent of the piston is generally very easy, one man can work the press by an injecting pump of larger dimensions. Large presses are generally furnished with two injecting pumps, a larger and smaller, standing side by side, and communicating in common with the forcing valve. If the large pump be a two-inch, that is, have its plunger two inches in diameter, and the smaller a one-inch, and their barrels be the same length, the quantity of water forced in every stroke of the larger is four times the quantity which is forced in every stroke of the smaller. Hence the larger pump raises the working piston four times as fast as the smaller. But since the pressure is proportional to the area, the same man, working at the same length of lever, can only raise one-fourth the load at the large pump that he can at the smaller. Hence the large pump is used, for the sake of speed, as long as it can be, and when the load becomes too great, it is relinquished, and the smaller one is worked.

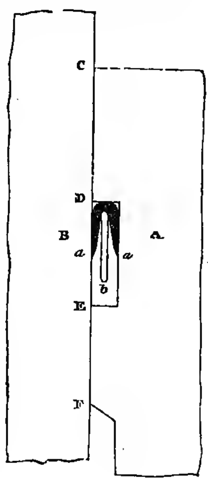
In the construction of this two-inch pump, an ingenious contrivance may be mentioned, for preventing accident or derangement in the machinery. Should the forcing valve be prevented from closing from any cause, as the intervention of dirt, immediately on the descent of the plunger, the action which would be transmitted from the water in the working cylinder to the base of the two-inch plunger might, from the workman not being aware of the fact and not having sufficient command over the pump-handle, raise the plunger too high, and force it entirely out of the barrel. To guard against this, a hole is drilled up the plunger, and another at right angles to it, as represented by the dotted line in the figure (Art. 160). Now when the pump is worked properly, the orifice of this horizontal hole does not rise above the top of the leathers,

but when the plunger is raised a little higher, the orifice rises above them, and the water, finding its way out between the metal surfaces of the plunger and barrel, the pressure on the base of the piston is immediately diminished.

158. The strength of materials has been shewn to be the only theoretical limit to the effort of the press, and the slow ascent of the piston has been mentioned as a practical limit, which defect is however partially obviated by the employment of a large and small pump, which are worked in succession. But the most serious practical limit to its enormous power arises from the extreme difficulty of keeping the piston and plunger water-tight. When the pressure exerted is more than three tons on the circular inch, the leathers wear out inconceivably fast, and new ones must be put in.

159. The contrivance by which the piston is rendered water-tight, is too ingenious, and too important a point in the history of the press to be omitted.

The accompanying figure represents a portion *A* of the working cylinder, and a portion *B* of the piston, whose surface *CDEF* is in contact with the cylinder at *CD* and *EF*. The distance *CF* is about nine inches, and *CD*, *DE*, *EF*, each three inches.

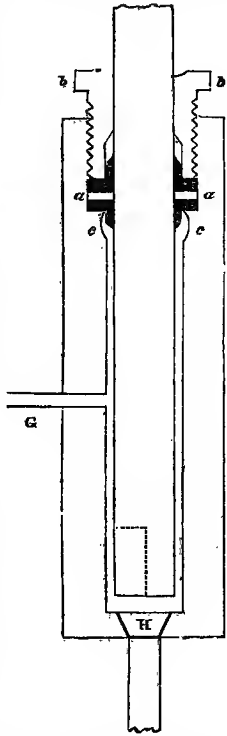


Opposite *DE* is a recess about half an inch deep, in which the leather collar is placed. This collar is a double leather turned over a metal ring; so that a vertical section of the collar and ring would appear as in the figure, the dark part representing the leather, and the light figure, with *b* below it, between the leather representing the ring.

Now the first water which is pumped into the cylinder insinuates itself between the metal surfaces at *FE*, and enters the recess, and acting on the under surfaces of the leather, forces them against the piston on one side, and the back of the recess on the other. Now as more water is forced in, and the piston begins to rise, the pressure on the inner surface of this leather increases, so that the piston and cylinder are tightly embraced all round by the leather collar, and no water can possibly insinuate itself at the edges *a*, *a*, so as to get behind the leather and escape. The greater the pressure the more closely is the leather pressed to the metal, so that if the leathers be sound, no leakage can possibly take place. The leather of course rises up to the top of the recess during the ascent of the piston; in fact, it will in general rise there by the action of the water before the piston begins to ascend; and when the discharge valve is opened for the descent of the piston, the pressure of the water being removed, the leather is no longer pressed tightly against the surfaces, but the leather and ring slide down with the piston till the ring touches the bottom of the recess. The ring is somewhat deeper than the leather, so that when the ring and leather come to the bottom of the recess, the edge *a* next the piston may not be drawn in between the piston and cylinder at *E*, which would draw the leather out, and stop the descent of the piston.

160. The construction of the injecting pump is equally worthy of attention. The accompanying figure represents the barrel and plunger piston, with the leather for keeping the pump water-tight, and the method of putting them in. The dark parts represent the leathers, which are in two pieces, with a copper ring *a a*, lying horizontally between them, having its upper and under surface scored, so that when the plunger is put in, and the piece *b b*, (with a cylindrical

hole turned accurately to fit the plunger) is screwed down upon the upper leather, the junction between the leathers and the ring is very complete, and the leathers are squeezed out against the piston on one side, and the barrel on the other, so that no water can pass them. The barrel is turned out, as represented at *c c*, that the water may get readily at the back of the leather, and, pressing it against the plunger, preclude the possibility of any being forced by, during the descent of the plunger.



161. *Practical applications.* This is perhaps the most perfect, and at the same time the most powerful machine with which we are acquainted. The great impediment to the perfection of all machinery is the friction; in this press it is scarcely worth considering; for when the surfaces of the piston are well polished and perfectly clean, the friction between their oiled surfaces and the leather collar is very small, and can scarcely be considered as any practical impediment, whereas the friction in the screw press under heavy pressures is an absolute barrier to its use, where the force of a single man is to be applied.

Another serious evil in machinery is, the rapid wear of metal surfaces in contact. But in this press, there are none exerting pressure on each other in contact. For the parts *CD*, *EF*, (Art. 159) are in contact only for the purpose of

insuring the steady and vertical ascent of the piston, and the wear on these is almost insensible.

New leathers are the only repairs which the press ever requires, and under ordinary pressures these will last many years. So that the press seems to possess in a very high degree the rare qualities of requiring no useless expenditure of force in the working, and very few repairs.

There is scarcely any department of the arts in which pressure is required, where this press is not used. A few of its applications will be mentioned.

It is used in packing, where large bulks are to be reduced into small compass, to be stowed on ship-board; large quantities of hay so reduced were exported for the use of the troops during the late war.

Some of the greatest pressures ordinarily required, occur in the extracting of the oil from hides, previous to their being tanned; of the moisture from paper; of the oily matters from tallow and from palm and cocoa-nut oils so as to separate the eleine from the stearine in the manufacture of candles; and in the manufacture of sugar.

One most material advantage which the press possesses is, that the cylinder may be placed any where, so that the table may be brought on a level with the floor, and the substances wheeled or rolled on to it. The injecting pump may also be at any distance from the working cylinder. An arrangement of this nature is adopted in the powder mills. It is necessary to subject the gunpowder, when in a moist state, previous to the granulation, to a great pressure, to squeeze out the water. This is attended with some danger; hence the press is set in one room, and the pump is at a considerable distance, with a strong wall between the press and the man who works the pump.

The most advantageous case in which the press can be applied, is, where an enormous pressure is required to be

exerted through a small space. Hence it is peculiarly adapted for trying the strength of cables, and masses of metal; for pulling up piles and trees; for sustaining or raising a building that has settled, that the under part may be restored; and for separating large masses of stratified rocks, as in the slate-quarries in North Wales.

The motion of the large piston may also be employed to actuate the wheel-work of a crane for raising weights, but this is not in general an advantageous application, by reason of the comparatively small space through which the piston moves.

CHAPTER XII.

ON TEMPERATURE AND HEAT.

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162. Matter has been divided into two classes, namely, solid and fluid, and some of the laws which regulate the state of rest and of motion in fluids, have been discussed in the preceding chapters. We have now to consider the effect of an agent of universal influence in nature, and one which is most intimately connected with the laws of the equilibrium and motion of fluids.

Two opposite actions or forces, distinguished by the terms attractive and repulsive, exist in nature, the former of which are exhibited in retaining the particles of bodies in apparent contact, and the latter in repelling, as it were, their approach; and the state of substances is determined in a great measure by the relative intensity of these forces. In solids, the force of attraction or cohesion is very powerful; in liquids, it scarcely exists at all, and in gases, it is absolutely insensible; and the particles of gases act on each other with a repulsive force.

Now it is a very general opinion, and supported by strong facts, that this repulsion is in some way or other connected with heat or caloric, this term being used to express the cause or principle of heat.

An inquiry of great importance, and to which much attention has been directed, is as to the quantity of sensible caloric which exists in any body, that is, the quantity or state of caloric which affects the senses, and gives rise to the sensations of heat and cold; which state is expressed by the word temperature.

We are now inquiring, not after the absolute quantity of caloric contained in any body, but how much the temperature of one body is greater or less than the temperature of another; and this is done most readily, by observing the relative effects which the temperature of these bodies has on some other substance which is affected by them. Now the bulk of all substances is affected by variations in the temperature, increasing with an increase of temperature, and decreasing with a decrease. And the slightest variation in the temperature produces very great increase or decrease in the bulk of some substances, as for instance, in the bulk of atmospheric air and of all the gases.

But liquids are better adapted for measuring variations in temperature than either gases or solids; for the variations in the bulk of the former are too great, and of the latter too small, to be generally useful. Another point to be attended to is, that the expansion and contraction of the substance used, must be regular and uniform. Mercury answers these purposes better on the whole than any other liquid. For it can be subjected to great heat and cold, without boiling and freezing, and its expansion and contraction between the freezing and boiling points are very regular and uniform. Its expansion and contraction are best observed by enclosing it in a tube, and an instrument of this kind is called a thermometer. We will now proceed to explain

The common Mercurial Thermometer.

163. Having taken a glass tube of about eight inches long and of very small but uniform bore, let one end be fused and blown into a bulb. Let the bulb and tube be heated so as to rarefy the air, then if the open end be immersed in mercury, as the air cools and contracts, the pressure of the atmosphere will force the mercury up the tube into the bulb. Should a sufficient quantity of

mercury to fill the bulb, and about a third of the tube, not be forced in, the operation may be repeated. The tube must then be hermetically sealed; this is effected by heating the mercury, so that it rises to the top of the tube and expels the air, and then darting the fine pointed flame from a blow pipe across the end of the tube, the glass is fused, and the orifice stopped, before the mercury has descended at all from the top of the tube.

The tube must now be graduated so that the temperatures indicated by different substances may be compared together, and that different thermometers may be compared together, two fixed and invariable points must be obtained, which shall be the same in every thermometer. The practice now universally followed was introduced by Newton, and is founded on the fact, that a thermometer plunged into ice that is thawing, stands at the same height in all countries; and also that under the same pressure of the atmosphere, a thermometer plunged into boiling water stands invariably at the same height. These two facts give the invariable points which are so necessary. The thermometer is placed in a vessel containing snow or pounded ice which is just thawing, and when the mercury in the tube is observed to be perfectly stationary, the point at which it stands is marked as the freezing point, or point of congelation.

Again, the thermometer is held in the vapour of boiling water, and the vapour being allowed to pass off, a fresh supply of vapour comes constantly in contact with the tube, and when the mercury has become perfectly stationary, the point at which it stands is marked as the boiling point.

The rise of the vapour is considerably affected by the pressure of the atmosphere (Art. 192). Hence to avoid all inaccuracy, some standard must be chosen, and the pressure when the barometer stands at 30 inches, is generally used in determining this point.

164. Prop. *To compare the graduation of Fahrenheit's, Reaumur's, and Celsus's, or the Centigrade Thermometer.*

The invariable points of freezing and boiling being fixed, the distance between them is differently divided in different thermometers.

In Fahrenheit's, the distance is divided into 180 equal parts, called degrees; the freezing point being 32^0, and the boiling point consequently 212^0.

In Reaumur's and the Centigrade, the freezing point is the zero of the scale; and the distance between the freezing and boiling points, is in Reaumur's divided into 80 equal parts, and in the Centigrade into 100.

The same distance is divided in the three thermometers, 180^0, 80^0, and 100^0. Hence, indicating the respective thermometers by the initials F, R, C, the length of a degree in each will be as follows:

$$1^0F. : 1^0R. : 1^0C. :: \frac{1}{180} : \frac{1}{80} : \frac{1}{100}$$

$$:: \frac{1}{9} : \frac{1}{4} : \frac{1}{5}.$$

But the temperature is measured by the number of divisions contained in equal portions of the stem of the respective thermometers. Now the zero point of Fahrenheit's is 32 degrees below freezing point. If therefore $F^0 - 32$, R^0, C^0, indicate the same temperature on each of the three thermometers, we have the proportion

$$F^0 - 32 : C^0 : R^0 :: 9 : 5 : 4,$$

whence result the following equations for passing from one scale to another.

$$4(F^0 - 32) = 9R^0, \quad 5(F^0 - 32) = 9C^0, \quad 5R^0 = 4C^0,$$

$$\text{or, } \frac{1}{9}(F^0 - 32) = \frac{1}{4}R^0 = \frac{1}{5}C^0.$$

The divisions principally used are those of Fahrenheit and the Centigrade, and the equations for passing from the indications of the Centigrade to those of Fahrenheit are

$$F^0 = 32 + \frac{9}{5}C^0, \quad C^0 = \frac{5}{9}(F^0 - 32),$$

that is, "add 32 to $\frac{9}{5}$ths of the number indicated on the Centigrade, and the result is the number which would be indicated by Fahrenheit," "subtract 32 from the number indicated by Fahrenheit, and $\frac{5}{9}$ths of the remainder is the number which would be indicated by the Centigrade."

165. The mercurial thermometer answers extremely well for all temperatures between freezing and boiling; for although mercury, like all other liquids, expands in an increasing ratio as the temperature increases, that is, the same quantity of heat causes a greater expansion of the mercury when the temperature is near the boiling point, than when it is near the freezing point, yet between these points the ratio of this increase is precisely the same as the increase of the glass; hence the increasing expansion of the tube and bulb compensates for the increased bulk of the mercury, so that no correction is requisite on this score. But for temperatures beyond the boiling point, glass expands more rapidly than mercury.

Hence though the mercurial thermometer may be made to indicate temperatures which exceed $212^0 F.$, or fall below zero, by continuing the division of the stem either way, yet since the variations are not so regular as between these points, and require some corrections, it does not answer equally well. Beyond 212^0, the expansion of glass increases in a more rapid ratio than the expansion of mercury, so that the real expansion is greater than the apparent. Add to which, mercury boils at about $660^0 F.$, and freezes at $-39^0 F.$, that is, at 39 degrees below zero.

Hence also it cannot be employed at all to measure greater degrees of heat or cold than these, and recourse must be had in these cases to the following methods.

The Pyrometer. When intense heat is to be measured, an instrument called the Pyrometer is used, which is made of a solid or gaseous substance.

Daniell's Pyrometer is the one in general use; it consists of a small rod of platinum 10½ inches long, which is placed in a tube of black lead or earthenware, and the difference between the expansion of the platinum stem and the enclosing tube, is indicated on a circular scale. One degree of this pyrometer corresponds to seven of Fahrenheit, and very high temperatures may be measured by it.

Alcohol Thermometer. When any intense cold is to be measured, a thermometer filled with spirits of wine, instead of mercury, is used. For spirits of wine or pure alcohol has hitherto retained its fluidity under every degree of cold to which it has been subjected.

166. *Register Thermometer.* For some purposes, especially in making meteorological observations, it is very desirable to be able to ascertain the highest and lowest temperatures which may have taken place during the absence of the observer. This is effected by two thermometers, one filled with mercury, and the other with alcohol. The bulb is bent at right angles to the stem, so that the graduated stem may conveniently be placed in a horizontal position.

A small cylindrical piece of black enamel is placed in the stem of such a size as to move freely in the tube.

When the mercurial thermometer is to be left, the enamel, by holding the stem vertically, is shaken down so as to touch the mercury. Then the stem being placed horizontally, as the mercury expands, the enamel floating on the mercury is pushed forward, and when the mercury

recedes, the enamel stays at the point to which it has been conveyed. Thus the highest temperature which has taken place during any interval is accurately known.

When the alcohol thermometer is to be left, the enamel is placed in the same manner in contact with the surface of the spirit, and as the alcohol contracts, the enamel recedes with it, owing to its adhesion to the spirit. When the alcohol expands again it readily passes beyond the enamel, which being of much greater specific gravity than the spirit, stays at the point to which it has been drawn.

167. *Difference between Temperature and Heat.* The temperature of a body may be defined to be its state with respect to heat, as indicated by the thermometer or by our feelings. The thermometer gives us no information respecting the quantity of caloric contained in any substance, it only gives us a measure of the same kind of knowledge, as that, which may be discovered by our feelings alone. The information which our feelings give us is, from causes which will be mentioned hereafter, necessarily very uncertain; but still, it is the same in kind, as the preceding. It must be remembered also, that these degrees of temperature are the divisions of an arbitrary scale, chosen at convenience, and without any reference to the absolute quantity of caloric which exists in bodies.

The term *heat* has two distinct meanings, being used to express a sensation which can only exist in a sentient being, and the unknown quality of bodies by which they excite in us that sensation. When used in the latter sense it is synonymous with the term *caloric*, which is employed exclusively to signify the quality of bodies which excites the sensation, or, the cause and agent of the effects which we see produced.

These observations, and the frequent reference which must be made to the agency and laws of heat for the

explanation of certain physical phenomena, render a brief explanation of the laws which have been made out in this branch of philosophy necessary for our present subject.

168. PROP. *Heat is an inseparable quality of matter, and is readily transferred from one body to another.*

No substance has been found absolutely devoid of heat, for any substance, however apparently so, may be made to cause a rise of the thermometer, and thus indicate the escape of heat. Thus heat is transferred from the substance to the thermometer, and this transfer takes place most readily and quickly. For if a cup of mercury at $60^0 F.$ be placed in water at $212^0 F.$, an equilibrium of temperature will soon be established, so that a thermometer will stand at the same height both in the mercury and in the water.

The varying sensations of heat and cold arise partly from a like cause. On touching a body whose temperature is different from the temperature of the hand, the equilibrium of temperature will soon be established, and the substance feels hot or cold, according as the temperature of the hand is being raised or lowered.

It would be foreign to our present purpose to enter upon the question as to the nature of heat or caloric; whether it be a subtle fluid emanating from hot bodies, and entering betwixt the particles of cold bodies, or whether it be a vibration or state of particles *.

169. PROP. *Heat is transferred by communication and radiation; the intensity of the communicated heat depends on the nature of the substances in contact, and the intensity of the radiated heat varies inversely as the square of the distance.*

The transfer of heat by communication may be distinguished into transfer by *conduction*, and transfer by *con-*

* See Webster's *Elements of Physics*, Art. 188.

*vection**; the former term being applied to the method of transfer which takes place in solid bodies, and the latter in fluids.

When a solid mass is placed in contact with a hot substance, the heat is communicated from the one to the other, and is transmitted through the solid, being *conducted* from one particle to another with a velocity which depends on the nature of the substance in contact; hence substances have been divided into good and bad conductors. Thus, a piece of metal is a very good conductor, and a piece of glass a bad conductor, and a piece of silk or fur a very bad conductor. The difference in the conducting power of different substances, serves to explain the different sensations of heat and cold which we experience on touching substances in which the thermometer indicates the same degree of temperature. Thus if the hand be laid on a piece of marble, on a table, and on the carpet, all of which being in the same room, will indicate the same degree of temperature, if the bulb of a thermometer be applied to them, the sensations will be extremely different; the marble is a good, and the carpet a bad conductor.

But when heat passes by communication from a hot body into fluids, the transfer takes place in a very different manner. A fluid becoming heated expands, and then being specifically lighter than that part of the surrounding fluid which has not received an increase of temperature, the particles ascend, and fresh particles descend into their places; and the transfer of heat by this motion of the particles is so rapid, that if one thermometer be placed at the top and another at the bottom of a vessel of fluid, which is heated from below, the upper thermometer will begin to rise almost as soon as the lower; this then is the transfer by *convection*.

* For this term, which was much wanted, we are indebted to Dr Prout, *Bridgewater Treatise*, p. 65.

The *conducting* power of fluids is extremely small. For if heat be applied to the upper surface of a fluid, this motion of the particles cannot take place, and consequently the only transfer of heat which can take place, will be by conduction, in consequence of the contact which subsists between the particles. And the transfer which takes place in this way is so small, that Count Rumford denied that water could *conduct* at all. But it appears that all fluids conduct heat in a slight degree.

Hence the transfer by communication, that is, by conduction and convection, depends on the nature of the substance.

The *radiation* of heat is that transfer which would take place *in vacuo*, and must be equal in all directions. Now the intensity of the heat is evidently less the greater the distance; and if the rays of heat diverge in right lines in every direction, a surface similarly situated with respect to every part of the body will be similarly affected. Hence if any number of spherical surfaces be conceived to be described about the hot body, each of these will be acted on by the same number of rays. And the surfaces being in the direct ratio of the squares of the distance, the number of rays, that is, the intensity of the heat is as the inverse square of the distance.

When a heated body is suspended in air, the cooling takes place, or the heat is transferred to the surrounding bodies, in all three ways. But the quantity which passes away by convection and radiation, is so much greater than that which is conducted from particle to particle, that the latter quantity cannot be calculated.

170. Prop. *Fadiated heat is reflected in right lines, the angle of reflection being equal to the angle of incidence.*

A small heated ball, or a ball containing hot water, was placed in the focus of a concave highly polished reflector,

the rays of heat radiated from the warm mass were reflected at the surface of this reflector, and received on another reflector in the focus of which the bulb of a small thermometer was placed; the thermometer indicated an increase of temperature.

From the position of these reflectors, it is evident, that if the rise of the thermometer were owing to the heat being twice reflected, then it must be reflected in right lines, the angles of incidence and reflection being equal. That this is really the case is made evident by using the differential thermometer.

This thermometer consists of a glass tube bent twice at right angles, and having a bulb at the end of each of the vertical stems.

The tube is nearly filled with sulphuric acid coloured with carmine, the bulbs being left full of air. Such an instrument cannot be affected by any change of temperature acting equally on both bulbs, but if there be the slightest variation in the temperature of either of the bulbs, the elasticity of the air in the one becoming greater than in the other, the liquid will be driven towards the bulb whose temperature is the lowest.

Now if the heat were transferred in any other manner than by reflected radiation, from the warm body to the thermometer, it would affect both bulbs equally. This however is not the case, for one bulb being placed in the focus of the second reflector, the liquid is driven into the other bulb, and the quantity of liquid which is driven in depends on the excess of temperature of the warm body above the temperature of the surrounding air.

171. Prop. *Those surfaces which reflect worst radiate best, and conversely; and the absorbing and radiating powers of any surface are equal.*

By observing in many various experiments the effect produced on his differential thermometer, Leslie discovered

that the nature of the surface of a warm body has more to do with its radiating power than the substance of the body. Smooth and polished metals radiate very imperfectly, but the same substances radiate very freely if the polish be destroyed by rubbing them with a file or scratching them, or by covering them with whiting or lamp black. But the reflecting power of smooth and polished surfaces appears to be diminished as their radiating power is increased.

Let a cubical vessel full of hot water, having one side bright and another covered with soot or lamp black, be set in the focus of one mirror, and the bulb of a differential thermometer in the focus of another; let the bright side be first turned to the mirror, and the thermometer be observed; then let the side which is covered with black be turned to the mirror and the thermometer observed, and it will be found that the effect produced on the thermometer in the latter case, as compared with the effect produced in the former, is as 100 to 12; or the black surface radiates at least 8 times as much as the bright surface. And from numerous similar experiments it appears, that the best reflectors are the worst radiators, and conversely.

Of the rays of heat which fall on any substance, some are reflected and others are absorbed; and it is evident that those which are not reflected must be absorbed. Hence the quantity absorbed is less in proportion as the quantity reflected is greater, and the same is the case with the quantity radiated. Since then it appears that the very same circumstances are favourable or adverse both to radiation and absorption, it is highly probable that these powers are equal. And this inference is strongly supported by experiments.

172. Prop. *When a body radiates more than it absorbs, its temperature falls; and when it absorbs more than it radiates, its temperature rises.*

This theory of Prevost serves to explain more facts than any other which has been started, and has therefore the

strongest claim to be received as the true one. He supposes that all bodies are constantly emitting rays of heat in all directions, and constantly receiving rays of heat from all directions. Consequently if two bodies be situated near each other, one of which is of a higher temperature than the other, the interchanges will be unequal, and these unequal changes having gone on till the two bodies are reduced to the same temperature, the rays emitted will be equal to the rays received, and therefore the temperature will remain constant.

Let a ball and a thermometer be placed in the foci of two reflectors. Then if the temperature of the ball be greater than that of the thermometer, the thermometer will rise, because it receives more rays than it emits. But if the temperature of the ball be lower than that of the thermometer, the thermometer falls, because it emits more than it receives.

This theory serves fully to explain the apparent radiation of cold. If a mass of ice or snow be placed in the focus of one mirror, and a delicate thermometer in the focus of the other, the thermometer will sink. Now, according to the theory of Prevost, this evidently ought to be the case, for the thermometer emits more than it receives, and consequently continues to sink until the equilibrium of temperature is established. When the ice is removed, the thermometer rises again, because then it receives more rays from the surrounding objects than it emits. A beautiful application of this theory takes place in explaining the formation of dew.

173. *Cooling of bodies.* It appears that the heat passes from a body by communication and radiation, and the rate of cooling depends on the nature of the surrounding substances. It is evident that a red-hot ball loses a great deal of heat during the first few instants, and that the

quantity lost each instant is much less, as the temperature of the ball approaches the temperature of the surrounding bodies. It was attempted by Newton to establish the rate of cooling; that is, the number of degrees of heat lost by a hot body during equal portions of time; and thence, to discover a law of cooling, that is, the relation which different rates of cooling bear to each other, according to the circumstances of the case. Newton thought that the quantities of heat lost in given small times bear a constant ratio to the excess of the temperature of the hot body above the surrounding bodies at the beginning of that time.

On this hypothesis, it will follow that the times being taken in arithmetic progression, the rates of cooling are in geometric; and the law thus deduced appears to hold very well for the cooling of bodies whose temperature does not exceed 212°; but for higher temperatures the rate of cooling is far greater than what the Newtonian law gives *.

The subject has been thoroughly investigated by Dulong and Petit, and the following facts have been established by them †.

When a body cools *in vacuo*, the heat which it loses is entirely owing to radiation. When it cools in any kind of air, the process goes on more rapidly, because the quantity of heat radiated is the same as *in vacuo*, and an additional quantity is transferred by convection (Art. 169).

The rate of cooling of a liquid contained in a vessel is not altered by the size nor by the shape of the vessel.

The celerity with which heat is communicated from hotter bodies to colder ones, all other circumstances being the same, depends on the extent of contact and closeness of communication between the substances. This last fact

* See *Theory of Fluids*, Art. 101, and Kelland's *Theory of Heat*, Arts. 36—43.

† See Thomson, *On Heat and Electricity*, p. 115.

is quite conformable to general experience, and explains why heat passes very slowly through bodies which are of a rough and spongy texture. Hence wood transmits heat more slowly than metals, cork more slowly than wood; and wool, feathers, and furs, more slowly still; hence these substances are used for clothing in cold weather. And the same substances will equally prevent heat from entering other bodies; thus a piece of ice may be kept from melting in a warm room by being well enveloped in fur.

On the Specific Heat of Bodies.

174. The thermometer gives no information whatever respecting the absolute quantity of caloric contained in, or communicated to any substance. For if two vessels of unequal size be filled with water from the same source, the thermometer will indicate the same temperature on being placed in either of them, whereas it is evident that the larger must contain a greater quantity of heating matter, that is, a greater quantity of caloric than the smaller. Also if a number of different bodies be so situated that they must receive equal quantities of caloric from some external source, it will be found that the heating effects upon each of them, as indicated by the thermometer, are not equal. If equal quantities of water are mixed together, one portion at 100^0, and the other at 50^0, the temperature of the mixture will be the arithmetic mean of the two temperatures, or 75^0. But if equal quantities of different substances be mixed together, the result is very different. If a pound of mercury at 160^0 and a pound of water at 40^0 be mixed together, the resulting temperature will be 45^0; but if the water had been at 160^0, and the mercury at 40^0, the temperature of the mixture would have been 155^0.

Thus it appears that the same quantity of heat, as indicated by the thermometer, gives 5 degrees to water, and 115 to mercury, which is in the ratio of 1 to 23. Hence,

in order to increase the temperature of equal weights of water and mercury to the same extent, the water will require 23 times more heat than the mercury; or if equal quantities of heat be added to equal weights of water and mercury, their relative increase of temperature will be expressed by the numbers 1 and 23.

All other substances are subject to a similar law, and the general proposition which may be considered as established is, *that equal quantities of different bodies require unequal quantities of heat to heat them equally*, or, *that different bodies have different capacities for heat.* Now it is necessary to have some term whereby to express this extraordinary quality of bodies, and the term *specific caloric* or *specific heat* has been adopted. The fact to be borne in mind in the application of this phrase is simply this, that every body requires a certain quantity of caloric to raise its temperature a certain number of degrees, and the quantity of caloric required to heat equal quantities of water and mercury one degree being in the ratio 23 to 1, the specific heats of water and mercury are expressed by these numbers.

It is very convenient to compare the specific heat of other substances with that of water. Hence if the specific heat of water be taken as unity, we have the specific heat of mercury

$$: \text{the specific heat of water } (=1) :: 1 : 23;$$

$$\therefore \text{ the specific heat of mercury} = \frac{1}{23} = \cdot 0434.$$

The experiments requisite for determining the specific heat of substances, and especially of gases, are extremely difficult, and the results very uncertain. The following circumstances are peculiarly worthy of notice*:

* Turner's *Chemistry*, pp. 53, 54.

(1) Every substance has a specific heat peculiar to itself, whence it follows, that a change of composition will be attended by a change of capacity for heat.

(2) The specific heat of a body varies with its form; a solid has a smaller capacity for heat than the same substance when in a state of liquid; the specific heat of water, for instance, being 9 in the solid and 10 in the liquid state.

(3) When any given weight of gas varies in density and volume under the same pressure, as when expanded by heat, its specific heat is unaltered.

(4) The specific heat of equal weights of the same gas changes with their density and elasticity. Thus when 100 measures of air expand by diminished pressure to 200 measures, its specific heat is increased: and when the same quantity of air is compressed into 50 measures, its specific heat is diminished; that is, the specific heat varies as the volume.

(5) The specific heat of a substance is greater at high than at low temperatures of the substance; this appears to be owing to their dilatation.

(6) Change in specific heat is always accompanied by a change in temperature; increase in the former being accompanied by diminution in the latter, and conversely. Thus when air, confined within a flaccid bladder, is suddenly dilated by means of an air-pump, a thermometer placed in it indicates diminished temperature. On the contrary, when air is compressed, the corresponding diminution of its specific heat gives rise to increase of temperature; and so much heat is evolved by sudden and forcible compression, that tinder may be kindled by it.

Theory of Sound. The change in the specific heat of gases consequent on a change in their density, and the elevation of temperature consequent on sudden compression, are of great importance in all inquiries respecting the

velocity of a transmitted vibration in an elastic fluid. I calculating the velocity of sound, theory gives too small a velocity, unless a correction be made for the change consequent on the increase of temperature*.

High pressure steam does not scald. By high pressure steam is meant steam which being raised from water of more than 212° *F.*, has an expansive force greater than the atmospheric pressure. If the hand be held in steam issuing from a vessel where water boils freely, it receives a severe scald; but if water be heated in a close vessel to a very high temperature, the hand may be held with impunity in steam rushing from water of such a temperature when the valve is opened.

Such steam possessing very high elastic force, expands instantly to many times its original volume; this great increase of volume is accompanied with a corresponding increase of specific heat and diminution of temperature. (Obs. 4 and 6.)

175. *The air becomes colder as we ascend.* The air receives heat both from the sun and the earth, but the greater quantity from the latter; and when it becomes heated it expands and rises so that there is a constant transfer of hot air from the lower to the upper strata; it would appear therefore that the upper parts of the atmosphere ought to be at least as hot if not hotter than the lower. But the contrary is known to be the case, and the temperature diminishes by about 1° for every 350 feet that we ascend. But the specific heat varies as the volume of a given quantity of gas (Art. 174, 4), hence when a quantity of atmospheric air is transferred to the upper regions, being increased in volume, its specific heat is increased; and if its specific heat is *increased* its temperature is *diminished* (Art. 174, 6). Hence it follows that the temperature of

* See *post,* Arts. 222—4, and *Theory of Fluids,* Art. 142.

the upper regions is less than that of the lower, and since the density of successive strata diminishes in geometric progression (Art. 119), the specific heat of each stratum will increase, and therefore its temperature will decrease.

176. *Level of perpetual snow or congelation.* The volume of a given portion of air will increase in a geometrical progression as it ascends, hence its temperature will fall rapidly, and therefore there will be above every place on the earth's surface some elevation at which the temperature is never above $32^0 F.$, which will be a point of perpetual congelation. The level on which this point is situated will be very different in different latitudes; and will depend partly on the general diminution of temperature as we travel from the equator polewards, but principally on local circumstances. Wherever an extensive tract of cultivated land occurs, the level of congelation will be much higher than is due to the latitude of the place, and where extensive glaciers exist on an uncultivated tract, it will be lower.

Latent Heat.

177. The singular facts stated in the preceding article, that substances of equal temperature contain unequal quantities of heat, and that in making a mixture of two different substances a certain quantity of heat becomes absolutely lost, or at least insensible, lead to the theory of latent heat.

Black supposed that heat exists in bodies in two opposite states; in one it is supposed to be in chemical combination exhibiting none of its ordinary characters, and remaining, as it were, concealed without evincing any signs of its presence; in the other, it is free and uncombined, passing readily from one substance to another, affecting the senses in its passage, determining the height of the thermometer, and in a word, giving rise to all the phenomena which are attributed to this active principle. The heat which exists

in the former state, Black calls *latent*, and in the latter *free*. Perhaps the terms *sensible* and *insensible* are preferable, since* they express the fact without any reference to the cause of it; and all we at present know is, that a certain quantity of heat which was sensible, that is, indicated by the thermometer, becomes insensible, or ceases to be indicated by it.

178. Prop. *During the process of liquefaction heat becomes insensible, and during the process of congelation it becomes sensible.*

The theory of latent heat was adopted by Black to explain the phenomena of liquefaction and vaporization, as exhibited in facts similar to the following.

If a pound of water at 32^0 be mixed with a pound of water at 172^0, the temperature of the mixture is intermediate or 102^0, thus one loses as much as the other gains. But if a pound of water at 172^0 be added to a pound of ice at 32^0, the ice will quickly dissolve, and on placing a thermometer in the mixture it will be found to stand not at 102^0 but at 32^0. Thus the pound of hot water actually loses 140 degrees of sensible heat, all which enters into the ice and causes its liquefaction, but without in any way affecting its temperature; thus it appears that a quantity of heat sufficient to raise a pound of water by 140 degrees becomes insensible during the melting of a pound of ice. This explains the well-known fact on which the graduation of the thermometer (Art. 163) depends, that the temperature of melting ice or snow never exceeds $32^0 F.$; the temperature makes a stand, and all the heat which is added becomes insensible till the liquefaction is complete. And not only does a quantity of heat become insensible during liquefaction, but a quantity becomes *sensible* during congelation.

* Turner's *Chemistry*, p. 47.

If when the air is at 22^0 a quantity of water in a tall glass be exposed to it, the water gradually cools down to 22^0 without freezing. It is therefore 10 degrees below the freezing point. If the water be then slightly shaken, part of it instantly freezes, and the temperature of the whole instantly rises to the freezing point; so that the water has acquired 10 degrees of heat in an instant. Thus it appears that the instant the water passed into the solid state a quantity of heat became sensible, which was before insensible.

It is also found that if a delicate thermometer be suspended above water in the act of freezing, it is affected by a stream of air of higher temperature than the surrounding air*.

The temperature of water in the act of freezing continues at 32^0 even when exposed to an atmosphere in which the thermometer is zero. Now if water under such circumstances is to preserve its temperature, heat must be supplied to it as fast as it is abstracted. Whence can it be supplied but from the heat which during the process of liquefaction became insensible?

179. Prop. *During the formation of vapour heat becomes insensible, and during its condensation heat becomes sensible.*

Several cylindrical tin vessels containing water were equally heated. The temperature of the water was 50^0, and in 4 minutes the water boiled, and in 20 minutes the whole had passed off in vapour. Now since the water was raised from 50^0 to 212^0 in 4 minutes, it follows that it received a quantity of heat each minute

$$=\frac{162^0}{4}=40{\cdot}5^0.$$

* See Thomson, *On Heat and Electricity*, p. 180.

Hence in 20 minutes it received a quantity of heat equal to 810^0, by which it was enabled to pass off in vapour; and yet the temperature of the steam was never above 212^0.

Again, water was heated in a strong close vessel called a digester to 400^0 *F*. On opening the valve about one-fifth of the water passed off in steam, and the temperature of the rest fell immediately to 212^0; thus the escape of a small quantity of vapour is accompanied with a great loss of sensible heat, which equals $188^0 \times 5$, or about 940^0.

When a quantity of steam is condensed into water, the heat again becomes sensible, and it appears from the experiments of Black and Watt, that steam of 212^0 in being condensed into water of 212^0, gives out as much heat as would raise the temperature of an equal weight of water by 950 degrees, all of which had been previously insensible to the thermometer.

From these two facts, it appears that steam is water combined in some way or other with about 945^0 of heat, the presence of which is not indicated by the thermometer.

180. *Fluidity the consequence of latent heat.* From numerous experiments, some of which have been mentioned, it appears that when a solid is converted into a liquid and liquid into vapour, a very great quantity of heat becomes insensible. Hence Black concluded that since this quantity of heat does not make the body apparently warmer, it must be thrown into it in order to convert it into a liquid; and that this great addition of heat is the principal and most immediate cause of the fluidity induced. It appears also that when a liquid assumes the form of a solid, a very great quantity of heat leaves it without sensibly diminishing the temperature; whence it may be concluded that this state of solidity cannot be induced without the abstraction of this great quantity of heat.

181. Before quitting this subject, a few phenomena of common occurrence and referable to these principles are worthy of notice.

Slaking of lime. When water is poured on fresh burnt lime, intense heat is evolved. Here water is passing into a solid state in consequence of its combining with the lime, and consequently, heat which was before insensible becomes sensible.

Mixture of sulphuric acid and water. In this case heat becomes sensible, but there is a diminished volume; a pint of each does not make a quart when mixed.

Freezing mixtures. The general theory of these mixtures is, that one if not all the component substances is solid, and when mixed they begin to liquefy. Thus heat becoming insensible as the substances pass from a solid to a liquid state, cold is generated.

Annealing. Most metals are both malleable and ductile. Now when metals have been hammered they become brittle, and their malleability can only be restored by heating them in the fire and cooling them slowly. This process is called annealing. But the point to be attended to is, that during the hammering of them they become hot, or heat becomes sensible, thus their brittleness appears to be occasioned by heat being forced out, and their previous qualities are restored by replacing this heat which has been forced out.

For the mathematical part of this most important subject the student is referred to the *Theory of Heat*, by Mr Kelland.

181 *a.* *Perkins's Hot Water Apparatus.* The laws of water in relation to heat are well illustrated by an apparatus known as Perkins's system of hot water apparatus for warming buildings, heating liquids, evaporating syrup of sugar, melting soap, and other purposes in which heat of

various degrees requiring to be regulated with great precision, is employed. A peculiar feature also of this system is the transfer of heat to a considerable distance from the generating source with the least practicable amount of loss. These objects are effected by means of water circulating in an hermetically closed circuit of pipe; the circulation being occasioned by the inequality of pressure introduced into the circuit by reason of the less specific gravity of the more heated portion of the water.

The furnace or source of heat consists of a coil of iron pipe*, within which the fire is placed, the pipe at the upper part of the coil rising up and connecting itself with other coils situated in the places to which the heat is transferred, and the lower part of the coil being connected with the return pipe from these coils. Suitable apparatus, as an expansion tube or a valve connected with a reservoir into which the water from the interior of the pipe may flow on its expansion, and from which the water may be supplied on its contraction, is placed in any convenient part of the circuit, generally in immediate connexion with the highest part of the circuit. These coils so connected together form one continuous pipe or circuit in which the heated water can circulate. The circuit of pipe having been filled with water (by means of a force-pump when necessary), and the

* The pipe employed is that known as Russell's (inch diameter) drawn iron tubing, which can be bent cold and turned in any direction. This pipe is manufactured in lengths of about 15 feet; and the manner in which these lengths are joined together so as to form one continuous metallic pipe, is especially deserving of notice. The two pieces to be joined must be brought together endwise, the one end being formed as a plain butt, and the other end being tapered off so as to present a chisel-edge; a right-hand screw is cut upon the end of one length, and a left-hand screw upon the end of the other length, and a screw-collar being placed betwixt them, the two pieces are brought together by turning this screw-collar until the chisel-end cuts into the plain butt-end, and thus forms a perfect metallic joint, as sound as any other part of the pipe.

aperture by which it was filled closed, the apparatus is complete. On the fire being lighted in the furnace or applied to the lower part of any coil, or to any part of the circuit presenting a vertical or ascending column, the water in the interior becomes heated and expands, the portion of the circuit so heated and expanded displaces a portion of the water above it, and having become lighter is unable to counterbalance the pressure of the colder column at its lower extremity, it consequently rises, and thus establishes a current throughout the whole circuit however long; as the temperature increases the rapidity of the circulation also increases, the water flowing away heated from the upper part of the coil and returning, after having parted with its excess of heat, to the lower part of the coil. This motion is continuous, and may in this respect be compared to the motion of the water in a siphon (Art. 144), or to an endless chain which receives continuous and uniform impulses as it passes a certain point.

It will be observed that this continuity of action results from the water being contained in an hermetically closed circuit; if the circuit were open to the atmosphere the water would be forced out, and the continuity of the chain, and consequently of the motion, destroyed. The pipe may be bent in any conceivable direction, and in warming rooms it may be carried over the tops of windows and doors, or under the floor, and adapted to any species of architectural embellishment. If the pipe at any part of its circuit should dip below the bottom of the furnace-coil, care must be taken to have a considerable excess of ascensional column, that is, of head of heated column arising from the upper part of the furnace-coil, otherwise the motion may not go on; hence in warming rooms on the same floor, supposing the furnace-coil to occupy the place of the ordinary grate, and that no considerable height of ascending column can be

had, it is expedient that the return pipe should not dip below the level of the bottom of the furnace-coil. It will generally happen that the furnace-coil is considerably below the level of the places to which the heat is to be transferred, as in the basement of the building to be warmed; in such cases the other coils may be placed in any convenient situations and in any number, all being connected together in succession at their upper or lower portions, as their respective levels may render most suitable for keeping the circuit as much as possible on one continuous ascent to its highest part, and on one continuous descent to its lowest part. The water so circulating in an hermetically closed circuit is constantly under pressure and in contact with the pipe; consequently no generaton of steam or deposit can take place, and the water may acquire extremely high degrees of temperature*. So great is the rapidity of the circulation, that the heat of the water may by clothing the pipe, be transferred without sensible diminution to a distance of many hundred feet; the heat is given out by the radiation of the pipe, and this *cæteris paribus* being in proportion to the surfaces exposed, the extent of surface or number of convolutions of the pipe constituting the coil at any particular place will depend on the heat required to be supplied at that place, and the cooling will go on subject to the laws which have been just explained (Art. 173). The coil of pipe from which the heat is to be given off, may be placed in a close vessel containing water, thus constituting a safety steam-boiler for generating steam, or at the bottom of a bath for heating the water to the required temperature, and for various other purposes in the arts; the requisite

* The temperature which the water can acquire will depend on the quantity of pipe outside the furnace, and on the furnace power or fire; by increasing the length of the circuit, the furnace or heating power being unaltered, the temperature of the contained water may be kept down to any convenient degree.

temperature when once ascertained for each particular apparatus being capable of regulation with the greatest nicety. The pipe, from its having the water always in close contact with it, and from the heated part of the circuit being rapidly swept away and replaced by the colder portion, is, practically speaking, almost indestructible. Another peculiar feature in this apparatus is the absence of any boiler; the circuit is of uniform diameter or section throughout, and each portion of the water is heated in succession as it passes through the part of the coil constituting the furnace. The tubing is so strong that it cannot be burst or rent under ordinary circumstances; and the phenomena observed when the pipe has been burst for the purpose of experiment are very curious. The pipe being of wrought iron yields or stretches until a slight rent or slit, generally imperceptible to the eye, takes places; from this rent a small quantity of water issues in the form of steam, which being of extremely high pressure is cold to the hand and passes at once into insensible vapour (Art. 174); the escape of that portion relieving the extreme pressure the circulation may go on as before. The rending of wrought iron tubes filled with water or steam when subjected to an intense heat in a furnace without the least danger to bye-standers, was the subject of many interesting experiments by Mr Perkins in generating steam for his steam-gun (Art. 190), when the attention of scientific men was directed to the use of steam for military purposes. The coil employed for warming an apartment produces this effect partly by radiation and partly by convection (Art. 169); the heat diffused by convection will be increased by introducing cold air to the lower part of the coil by a pipe communicating with the external air; by this means also the ventilation may be assisted, as the fresh air so introduced will displace

other air, and produce a change in the atmosphere of the building*.

* This invention is the subject of letters patent, granted originally in 1831, and extended in 1845 by the Privy Council to the inventor, Mr A. M. Perkins, of Francis Street, Gray's-Inn Road, London, to whom the scientific world is so much indebted for researches, in conjunction with his father, upon the compressibility of water, and the properties of highly elastic steam, and steam surcharged with heat. Slow as the progress and introduction of inventions involving the application of any of the laws of nature, and opposed to existing prejudices and interests, have generally been, it cannot but be matter of surprise that this invention should hardly be known beyond a few public buildings of the metropolis, and the mansions of a few of the nobility, whereas the invention is exhibited in its most useful form when employed to warm the hall and passages of an ordinary house; the grate in any room conveniently situated being replaced by a basket or coil of the pipe. The author gladly avails himself of this opportunity of bearing testimony from actual experience in his own house and chambers to the efficiency, cleanliness, and economy of this system under circumstances in which no other could have been readily practised. In the earlier periods of this invention it was not uncommon to dwell on the danger of water heated to a high temperature in close vessels; a representation perfectly true when the water is contained in large vsesels, as steam-boilers, in which steam can be generated, but untrue when applied to the apparatus of Mr Perkins.

The above apparatus presents very convenient means for verifying on a large scale the laws of convection and radiation. The heat of the pipe being observed at different parts of the circuit by suitable thermometers, many important practical conclusions as to the rate of cooling in different states of the atmosphere and under different circumstances might be obtained.

The heat transfused by conduction through the metal of the pipe in this apparatus may be disregarded, the pipe participating throughout the entire circuit in the temperature of the water in the interior.

CHAPTER XIII.

ON STEAM AND ITS APPLICATIONS.

182. When the temperature of water is raised to a certain point, the liquid passes into steam, which is an elastic vapour capable of re-assuming the liquid form on its temperature being diminished.

The temperature at which water passes into and is retained in the form of steam, is called the boiling point, being the point of elevation of the thermometer at which ebullition commences. This is a variable point for different liquids, and also for the same liquid, when the atmospheric pressure on its surface varies. This fact is easily established by observing the heights of the same thermometer placed successively in the same liquid, which exhibits the phenomena of ebullition when placed under the receiver of an air-pump, so that the atmospheric pressure is removed from its surface, and when in its natural state subject to this pressure.

It appears that water boils *in vacuo* at 72^0 *F.*, thus exhibiting the phenomena of ebullition at 140 degrees lower than in the open air.

Now the weight of the atmospheric column is subject to considerable variation, so that the boiling point of water examined by the same thermometer at different times will be different. Hence in the graduation of the thermometer, and in all accurate observations connected with the boiling point of liquids, the height of the barometer must be observed. When the barometer is at 30 inches, the pressure of the atmosphere is 15 lbs. on the square inch (Art. 125). This

height is generally taken as a convenient standard whereby to compare different observations.

The ratio between the depression of the boiling point, and the diminution of the atmospheric pressure is so exact, that Wollaston applied it to determining the height of mountains. A depression of 1^0 *F.* corresponds to an elevation of about 530 feet.

183. Prop. *The elastic force of steam increases at a greater rate than the temperature at which it is generated.*

Steam is water combined with a quantity of heat which corresponds to about 950 degrees of Fahrenheit (Art. 179); and steam is generated so soon as the expansive force communicated to water by heat, is equal to the pressure to which the surface of the water is subjected.

Hence, under ordinary circumstances, water cannot be heated to more than 212^0 *F.*; but when it is confined in a strong vessel, vapour collecting above its surface checks ebullition by its pressure on the surface, and the water may be heated to any temperature; but the pressure exerted under these circumstances on the sides of the vessel will soon be sufficient to burst any vessel.

The rapid increase in the pressure exerted on the sides of the vessel in this experiment, depends principally on the increase in the density of the steam which is collected above the surface of the water. But, since when steam is separated from water it possesses, so long as it retains its gaseous form, the properties of an elastic fluid, and follows for every increment of temperature the law of elasticity and expansion* (Arts. 89, 92), and though some addition to the pressure arises from this source, the rapid increase in the elastic force of steam, when the temperature of water in a close vessel is raised, depends prin-

* Gay-Lussac, *Ann. de Chimie*, Vol. XLIII.

cipally on the increase of the density of the steam, each accession of temperature causing a fresh portion of steam to rise, which adds its elastic force to that of the vapour already existing *.

Now the elastic force of steam at 212^0, is the same as the pressure of the atmosphere, or it will sustain about 30 inches of mercury; and the elastic force of steam at 250^0 will sustain about 60 inches; the increase of temperature equal to 38^0 *F.* doubles the elastic force of steam; and from what has been said, this steam may be taken as nearly double the density of steam at 212^0. A small part of the increase in the elastic force will be due to the increase in temperature, but it will be only a small part.

The elastic force of steam at different temperatures has been frequently examined, and tables have been constructed by Watt, Dalton, and more recently by Dulong and Arago †, all of which give nearly the same rate of increase; and it appears, that if the atmospheric pressure, or the elasticity of steam at 212^0 *F.* be taken as unity, the elastic force of steam at 234^0 *F.* will be about $1\frac{1}{2}$, and at 250^0 *F.* will be 2: thus an increase in the quantity of heat, indicated by 38^0 *F.* doubles the elastic force; and it also appears, that a further increase of 25 degrees trebles the elastic force, and a further of 18 degrees quadruples it. Thus, steam at 293^0 has 4 times the elastic force of steam at 212^0; and it is also found that steam at 510^0 would be 50 times stronger, that is, would sustain, 50×30, or 1500 inches, or 125 feet of mercury, or exert a pressure equal to 15×50 or 750 lbs. on every square inch. Thus the elastic force evidently increases much more rapidly than the temperature.

* Sharpe, *Manchester Memoirs*, Second Series, Vol. II.

† *Annales de Chimie*, Second Series, Vol. XLIII. p. 374.

A table of the elastic force of steam will be given at the end of the volume.

183*a*. Prop. *The sum of the latent and sensible heat of steam is a constant quantity.*

It appears from experiments that whatever be the temperature of steam from 212⁰ upwards, the heat given out, or the quantity of heat which becomes sensible on the condensation of a cubic foot of steam is the same; and this singular fact is expressed by saying that the sum of the latent and sensible heat is a constant quantity. Taking then the sensible heat of steam at 212^0 *F.* and the latent at 950^0 *F.*, we have 1162^0 *F.* as the sum of the latent and sensible heat of steam; the latent heat then of steam of any temperature will be known by subtracting the sensible heat from this constant quantity or 1162^0. Vapours of other liquids than water, and of a temperature below 212^0 *F.*, appear to exhibit the same remarkable condition in reference to the sum of the latent and sensible heat*.

184. *Generation and Application of Steam.* The practical application of steam involves two distinct questions, the one, how to generate the greatest quantity of steam of a given elastic force with the least quantity of fuel, the other, how to apply that steam most economically as a motive power. Through the medium of steam and the steam-engine, the active principles of fuel, or the power resident therein, is transformed to other elements, and applied to the production of motion. The quantity of steam generated, or of water evaporated, will depend on the fuel employed, on the manner in which it is burnt, and on the arrangements

* See on the *Heat of Vapours, and on Astronomical Refraction*, by Sir J. W. Lubbock, Bart., as to the accuracy of this law, and as to whether the true law may not be that the latent heat is constant, but that the quantity of fuel required to generate the latent heat is much greater than that required to generate the sensible heat.

for husbanding and economically applying the heat produced by the combustion of the fuel. The importance of economy in the application of fuel for the generation of steam, has given rise to various constructions of boilers, upon the details of which we cannot here enter, but recourse must be had to some of the numerous practical works on steam-engines and boilers.

Water is, as we have seen, converted into and exists in the state of steam in virtue of its latent heat; if then a sufficient quantity of heat be withdrawn, the steam assumes again the liquid form. If a close vessel be full of steam, and a jet of cold water be thrown in, the steam will be immediately converted into water. This operation is termed the condensation of steam, and from the facility with which it is performed may be most conveniently employed to create a vacuum, the creation of which is the fundamental principle of the steam-engine.

The various ways in which steam may be employed as a mechanical agent are exemplified in different constructions of steam-engines, which may be divided into two classes, *condensing engines* and *non-condensing engines.* Under the condensing engines will have to be considered *

(1) *The atmospheric engine*, in which the pressure of the atmosphere is made available by steam.

(2) *The single-acting engine*, in which a partial vacuum being created by condensation on one side of the piston, the elastic force of steam is available on the other.

(3) *The double-acting engine,* in which a partial vacuum

* It has not been thought necessary to advert here to the suggestions of the Marquis of Worcester as to the power of steam to raise water by its pressure, or to the engine of Savery, in which the pressure and condensation of steam in a vessel were employed to raise water; these being now matters only of historical curiosity, and will be found in most works on the steam-engine. See Tredgold *On the Steam-Engine*, 4to.

being created by condensation on both sides of the piston in succession, the elastic force of steam is available on the other.

In these engines steam may be used of the same elasticity as atmospheric air or steam at the temperature of 212°; but in the last two kinds of engines it is convenient to use steam of a little greater elasticity than the atmosphere.

These engines are also called low-pressure engines. And under the non-condensing engines comes

(4) *The high-pressure engine,* in which there is generally no condensation, but steam of high elastic power *must* be used.

The Atmospheric Engine.

185. A large vertical cylinder has a piston fitted air-tight in it, which is to be worked by the atmospheric pressure, rendered available by the application of steam. Now the piston is slightly overbalanced; that is, the piston-rod being attached to one end of a beam, which is poised on its centre, the other end is loaded with counterbalance weights a little heavier than the resistance to be overcome when the piston is moved from the bottom to the top of the cylinder; the resistance to be overcome being the weight of the piston and rod, the friction in the cylinder and the inertia of the mass to be put in motion.

The piston being slightly overbalanced would rise to the top of the cylinder if it were not for the pressure of the atmosphere on its surface; if then there be supplied to the under side of the piston a pressure equal to the atmospheric pressure, the piston will be enabled to rise. This pressure is so supplied by the admission of steam of 212° *F.* to the under surface of the piston. An equality of pressure being then produced, the counterpoising weight raises the piston to the top of the cylinder.

The piston is now at the top of the cylinder, and the lower part of the cylinder is full of steam; if a jet of cold

water be thrown into the cylinder, the steam will instantly be condensed, a vacuum will be created, and the pressure being thus removed from the under side of the piston, the atmospheric pressure will force it down with a pressure equal to about 15 lbs. (Art. 125) on every square inch of its area.

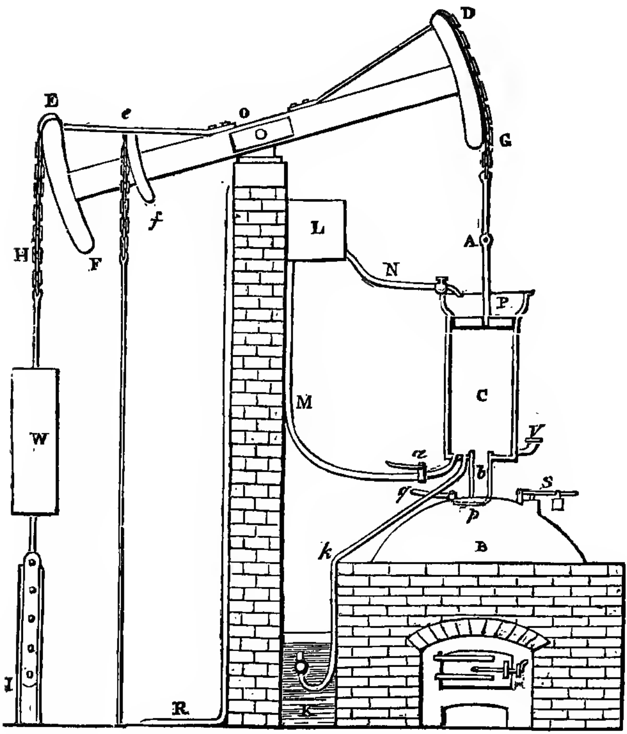
The water arising from the condensation escapes through a valve or pipe at the bottom of the cylinder just as the piston reaches the bottom. The piston being now at the bottom, steam is admitted again beneath it, and the operation repeated; this is the method of procuring moving force by the atmospheric engine.

The moving force generated during the descent of the piston raises the weight at the other end of the beam, and when the engine is employed for pumping water from mines, for which object it was invented, the rods of the pump-pistons connected with that extremity of the beam which is raised by the descent of the piston in the steam-cylinder, work the pumps on the return stroke or ascent of the piston.

The annexed diagram represents the atmospheric steam-engine as constructed by Newcomen, who was the first to employ a piston worked in a cylinder and connected to a beam; this diagram will also serve to illustrate the way in which the beam is employed, in stationary engines generally, whether atmospheric, single, or double-acting.

A is the piston-rod connected with the beam *ED*; *B* the boiler for generating and supplying steam to the cylinder *C* through a short steam-pipe *b*, which can be closed at the bottom by a slide plate *p* worked by the handle *q* so as to regulate as required the quantity of steam admitted into the piston; *S* the safety-valve for regulating the pressure of the steam in the boiler. The beam *ED* is balanced on and moveable about its centre *O*, and furnished with an arched head *DG*, by which it is drawn down as the piston descends, and an arched head *EF* for raising the pump-rod *I*,

and counterbalance weight *W*; another arched piece as *ef* may be employed for working a pump for the supply of condensing water to the cistern *L* by the pipe *R*. From the cistern *L* water is supplied by the pipe *N* for keeping the top of the piston *P* air and steam-tight, and by the pipe *M* for condensing the steam in the cylinder. *K* is the hot well into which the water passes by the pipe *k*, called the eduction pipe, which has a valve at its lower extremity for letting out the water from the steam-cylinder. *V* is a valve called a snifting valve for blowing out the air on the descent of the piston.



The diagram represents the piston at the top of the cylinder, where it has been brought by the weight of the pump-rod and counterbalance weight; the cylinder *C* having been filled with steam of atmospheric pressure, the further admission of steam from the boiler is cut off by closing the pipe *b* with the plate *p* moved by the handle *q*. The cock *a* in the pipe *M* is then opened and cold water injected into the cylinder from the reservoir *L*; the steam in the cylinder is condensed, the piston descends drawing down the end *DG* of the beam, and raising the pump-rods and weight connected with the other end at *E* and *e*; the water injected into the cylinder, and the condensed steam pass down the pipe *k* into the hot well *K*, and any air which had come into the cylinder with the injection-water or otherwise, is forced out through the snifting-valve *V* as the piston reaches the bottom of the cylinder. The piston having come to the bottom of the cylinder, the steam is let into the cylinder from the boiler to balance the atmospheric pressure above the piston, and the weights attached at *E* and *e* raise the piston and work the pumps. These operations, thus continued, constitute the working of the atmospheric engine. The opening and shutting of the steam-pipe *b* and cock *a* was at first done by hand, but a boy left in charge of them soon found the means of connecting them by strings with the moving parts, in order that he might indulge himself in play; and thus the first step was taken towards making the machine self-acting.

The piston and pump-rods are shewn connected with the ends of the beam by chains, which answer perfectly well in the atmospheric engine when the only action on the beam is downwards, but in the single and double-acting engines, rigid rods and parallel motions to preserve the vertical motion of the parts, notwithstanding the beam's motion about a centre, must be resorted to.

186. *Imperfections of the Atmospheric Engine.* When the steam has filled the cylinder, so as to balance the atmospheric pressure, and allow the piston to be drawn up by the counterpoise at the other end of the beam, the cylinder will have the same temperature as the steam; that is, will not be less than 212^0. Now when the jet of cold water enters, the steam becomes hot water at the bottom of the cylinder. But this water, not being under the atmospheric pressure, boils at a very low temperature, and consequently itself produces a vapour which would impede the descent of the piston. Add to which, the heat of the cylinder being 212^0, assists this generation of vapour below the piston; and in order that a sufficient vacuum may be produced, a quantity of water must be thrown in sufficient to cool the cylinder and steam to less than 100^0 *F.* And before the piston can rise again, the cylinder must be raised to 212^0, and while this is taking place, the steam is parting with its caloric in effecting it, and is reduced to water. So that at every descent of the piston the cylinder is cooled down to below 100^0, and every ascent has to be raised to 212^0. It is a question therefore whether the power gained by the increased perfection of the vacuum compensates for the increased consumption of fuel; and it is found, on the whole, more economical not to cool the cylinder to so low a temperature, but to work with a more imperfect vacuum, and consequently with a diminished power. The atmospheric engine is involved in this dilemma, either much or little water must be injected into the cylinder; if much, the vacuum is more perfect, and consequently the power obtained greater; but then the waste of steam, and consequently of fuel, is enormous; if little, the vacuum being imperfect, the power obtained is much smaller; but then the waste of steam and fuel is smaller also. One great problem therefore which presented itself to Watt was to create a vacuum without cooling the cylinder, and this he effected by his single-acting engine.

The Single-acting Engine.

187. We may arrange the different parts of this engine as three steps, suggested by difficulties which presented themselves. The first step was in the idea of producing a vacuum without cooling the cylinder. This led to condensation in a separate vessel. When the piston is at the top of the cylinder, a communication is opened into a vessel immersed in cold water, and in which there is a constant vacuum. This communication being opened, the steam rushes into the vacuum; and being instantly condensed, a vacuum is created in the cylinder beneath the piston, without lowering the temperature of the cylinder.

Thus the action goes on, without any waste of steam from the cooling of the cylinder.

But the condenser will soon become full of water from the condensed steam, and the vapour rising from the hot water would fill the condenser and render the vacuum very imperfect. The condenser must therefore be kept empty, and this led to the second step.

Watt made the condenser communicate with a pump, called the Air-Pump, which is attached to the beam (Art. 185), and makes a stroke every ascent and descent of the piston or beam, and draws off the water or vapour collected in the condenser.

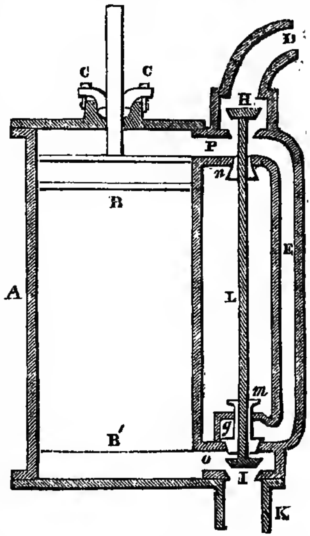
But the cylinder being open, the cold air coming in contact with its interior during the descent of the piston, reduces its temperature considerably. There was still therefore some waste of steam. This led to the third step, the closing of the cylinder, and making the piston-rod work through an air-tight stuffing-box; and the piston, instead of being pressed down by the atmosphere, was pressed down by the elastic force of steam of the same pressure as the atmosphere; that is, by steam of about 212^0.

By this last step all the principal causes of waste were removed; the external air was entirely excluded from the cylinder, and the piston was pressed down by the elastic force of steam, a vacuum having been created below it by condensation in a separate vessel. The external surface of the cylinder, being in contact with the external air, was liable to affect the temperature of the steam; and to exclude as much as possible waste from this source, the inner cylinder was surrounded by another, termed its jacket, and the intermediate space kept full of steam.

Another important alteration was made in the quantity of steam used; the piston moved with an accelerated motion, owing to the action of the steam continuing throughout the whole motion of the piston. To prevent this, and render the motion as uniform as possible, the steam is cut off; that is, its entrance is stopped on the piston having performed part of the stroke, and the remainder of the stroke is effected by the expansive power of the steam already admitted into the cylinder. Thus, during the last portion of the descent or ascent, the piston is urged by a force gradually decreasing, since the elastic force is inversely as the space occupied (Art. 89); and such an arrangement is found in practice to insure a uniform motion. Thus, some steam, and consequently some portion of the fuel and of the expense of working this engine, are saved, and a better motion, because a more uniform one, is obtained.

Besides the air-pump, there are two others; namely, a hot-water pump and a cold-water pump also worked from the beam as it vibrates. The water pumped from the condenser is discharged into the hot well, from which the boiler is kept supplied by the hot-water pump. The cistern in which the condenser is immersed would become hot, and water must therefore be supplied to it by the cold-water pump.

The annexed diagram shews the peculiar arrangement adapted for effecting the above purposes. *A* is the cylinder: *B* the piston: *CC* the stuffing-box filled with hemp moistened with tallow, having its cap screwed firmly down by two screws so as to form a steam-tight joint round the piston-rod. The piston-rod being turned true and polished is capable of moving up and down through the steam-tight joint. *D* is the steam-pipe leading from the boiler: *K* the eduction-pipe leading to the condenser: *H* and *I* are two valves upon the rod *L* which passes through stuffing-boxes or steam-tight joints at *m* and *n*. By raising and lowering the rod *L*, the valves *H* and *I* open or close their respective apertures in *D* and *K*. By means of a valve *g*, a communication between the top and bottom of the cylinder through the pipe *PE*, at *P* and *o*, may be made; the spindle of the valve *g* is hollow, and works in a steam-tight collar at *m*, and has the rod *L* passing through it, likewise steam-tight, in such a manner that the rod *L* with its valves may be moved independently of the valve *g*, or the valve *g* independently of the rod *L*. If the valves are placed as represented, *I* and *H* open and *g* shut, and the steam be permitted to come from the boiler through *D*, it will pass through *H* and enter the top of the cylinder at *P*, depressing the piston *B* by its elastic force, whilst the lower part of the cylinder is open through *o* and *K* to the condenser. When the piston is



depressed to B' the rod L must be moved downwards, and the valves H and I closed and the valve g opened, that the steam by its elastic force may pass through PE and the valve g, and act equally upon the lower as well as the upper surface of the piston: the piston therefore being in a state of equilibrium, as regards the pressure of the steam, will again be raised to its original situation by the counterpoise weight acting at the other end of the beam, and the steam will pass from above to below the piston. When it has arrived at that place, the valves may be again put in the first position, as represented in the drawing, when that at I being open, affords a communication through oK for the steam which now occupies the cylinder below the piston to pass to the condenser, whilst fresh steam from D presses upon the upper surface of the piston with a pressure equal to that of the atmosphere.

The single-acting engine is used principally for pumping; and one of the most interesting applications of it is in the Cornish mines, where, from certain peculiarities in the mode of working, the single-acting engine so applied is designated the *Cornish Engine,* to which we shall return hereafter.

The Double-acting Engine.

188. Both in the atmospheric and in the single-acting engine the steam is employed to produce motion only during the descent of the piston, and the weights or counterpoises which lifted the piston in its ascent act against the power in its descent. This action of the steam in one direction only and the corresponding intermission of the power is of no consequence when the engine is employed *directly* for pumping, which was the purpose for which the steam-engine was invented. But if machinery is to be driven,

a constant and uniform action is required; an unequal or an intermitted action being injurious to delicate machinery, and very inconvenient.

The idea occurred to Watt of making the steam press the piston up as well as down. This was readily accomplished by opening alternate communications, by valves, between each end of the cylinder and the boiler and condenser. Here as there is no loss of power from counterpoises, an equal vacuum being produced successively on each side of the piston, an equal moving power is obtained both on the ascent and descent of the piston. The steam may be cut off as before, after the piston has traversed a portion of the cylinder, and the motion communicated to the machinery rendered constant and uniform.

The nature of the arrangements of valves and steam passages which will be neccessary to effect proper communications for the admission of the steam to the upper and under side of the piston, and for its contemporaneous escape into the condenser from the under and upper side of the piston, will be understood from the diagram in the last article. Upon the construction, arrangement, and manner of working these valves, great mechanical skill has been expended; for an account of which recourse must be had to works on the steam-engine.

The engine usually employed in steam-boats, or, as it is commonly called, the *marine engine*, is a double-acting engine differing only from the engines as employed in land or stationary engines, in the the arrangements for converting and applying the motion of the pistons to the required purposes.

The steam used in the single and double-acting engines is that produced from water under a pressure a little greater than the atmospheric pressure. Hence the power obtained

does not result entirely from the vacuum which is produced on the opposite side at which the steam is admitted, but in a small degree also from the elastic force of the steam being greater than the elastic force or pressure of the atmosphere.

The High-pressure Engine.

189. In the engines already described, the power depends principally upon the vacuum obtained by means of the condensing apparatus. But the condensing apparatus requires the working of two pumps, namely, the air-pump and the cold-water pump exclusively for itself; and the apparatus for this purpose is an expensive incumbrance to an engine, rendered however in many cases very economical by the cost of fuel for the generation of steam.

If this apparatus be dispensed with, the engine will have its full moving force, excepting only the small force requisite to work the hot-water pump for supplying the boilers, capable of being applied to machinery.

We have seen that water under pressure may be heated to very high degrees of temperature, and the steam produced from such water possesses an enormous elastic force. Now if the condensing apparatus be not used, but the steam be simply allowed to escape into the atmosphere at the end of the stroke, the piston will always be resisted by a force equal to the atmospheric pressure, and the only part of the elastic force of the steam which will be available as a moving power, is the *excess* of its elastic force above the elastic force or pressure of the atmosphere. Hence, if steam of much greater elastic force than the atmosphere be used, the condensing apparatus may be dispensed with. Those engines which work with highly elastic steam, and without the condensing apparatus, are termed high-pressure

engines. The safety valves of their boilers are loaded with from 60 to 80 lbs. on the square inch.

Locomotive Engine. The locomotive engine on railways is one of the commonest illustrations of the high-pressure engine without any condensing apparatus. Steam of high elastic power, about 80 lbs. pressure on the square inch, is employed to propel the pistons, the rods of which by their action on the crank of the axle to which the wheels are fastened, causes the wheels to roll along the rails, provided the adhesion between the surfaces be sufficient.

Rotatory Engines. The class of engines called rotatory, because the whole steam is applied directly to drive round a wheel, or to produce circular motion, instead of on a piston in a cylinder, to produce rectilinear motion which has afterwards to be converted into a rotatory motion, furnishes another instance of high-pressure engines; steam of much greater elastic force than 14 lbs. to the square inch being usually employed in these engines. In some cases, however, atmospheric steam and a condensing apparatus have been employed in rotatory engines.

Reaction Engines. High-pressure steam issuing from the orifice of pipes fixed as arms radiating from a centre about which they can revolve, and bent or curved at their extremities, so as to cause the arms to move round by the reaction of the issuing steam at the orifices where the steam is dissipated into the air, is an instance of a use of highly elastic steam under circumstances in which no condensing apparatus could be of any avail.

The distinction between high and low-pressure engines must be regarded rather as speculative than as practical, since the steam actually employed has in most cases an elastic force far exceeding the atmospheric pressure; but the distinction between condensing and non-condensing engines is sound and practical, and the adoption of one or

the other rests on commercial considerations, entirely apart from any prejudice against or in favour of high-pressure steam.

The Cornish Engine.

190. The Cornish engine presents an instance of the application of steam so different from any of those already described, that it may almost be said to differ in kind from other steam-engines.

The power of highly elastic steam to generate momentum is well illustrated by the *steam-gun* (Art. 191), the invention of Mr Perkins, who applied highly elastic steam to impel bullets along a barrel and project them against an iron target at the distance of 30 or 40 yards. The momentum generated in the bullets by the continuous action of the steam during their passage through the barrel is so great, that they reach the target with sufficient velocity to be flattened by their impact on the target.

In the case of the bullet a great velocity is generated in a small mass; in the case of the Cornish engine a small velocity is generated in a large mass; the result to be regarded is the same in both cases, namely, momentum generated by the elastic force of high-pressure steam.

According to the system of pumping as practised in the Cornish mines, the object is to raise a great mass of pump-rods weighing several hundred tons, which weight on descending works the pumps. This enormous weight is suspended at one end of the beam of a single-acting engine, and is to be raised by the action of the steam on the piston. The cylinders and pistons are of great diameter, in some cases nearly 7 feet, and the length of the stroke from 10 to 11 feet *. Steam of from 60 to 90 lbs. pressure on the square inch is employed, and being let suddenly on

* The diameter of the Huel Towan and Fowey Consols engine is 80 inches, and length of stroke 120 and 124 inches respectively.

to the upper side of the piston, starts or puts in motion the enormous weight at the opposite end of the beam; the piston having begun to move, the steam continues to be admitted until the piston has travelled from about one-fifth to one-third the length of the cylinder, when it is altogether cut off; and the remainder of the stroke is performed by the elastic force of the expanded steam in the cylinder and the momentum generated on the first starting of the piston. When the piston has arrived near the completion of its stroke, a communication is made between the upper and under side of the piston by opening what is called the *equilibrium* valve*; the steam then pressing equally on both sides of the piston, the motion of the piston is speedily arrested, the down stroke is terminated, and the piston is for an instant stationary, that is to say, until the return stroke commences. The return stroke is effected by the weight of the pump-rods alone, the Cornish engine in this respect resembling the atmospheric and single-acting engine; the piston ascends not in all cases uniformly, but by a motion adapting itself to the descent of the pump-rods; the equilibrium valve or communication between the two sides of the piston remains open until shortly before the piston reaches the top of the cylinder, when that valve is closed, and all exit for the steam being cut off, it becomes compressed between the piston and cylinder-cover, and when the elasticity of the steam so compressed attains a pressure equal to the weight upon the other end of the beam, the ascent of the piston or up stroke terminates. At this instant the steam is let out from the boiler to the upper side of the piston, and the down stroke is made as already described.

On the equilibrium valve closing before the termination of the up stroke, a communication is opened between the

* This is a valve similar to *g* described in Art. 187.

bottom of the cylinder and the condenser, so as to make the vacuum as complete as possible on the under side of the piston during the down stroke, and until the equilibrium valve is opened to effect the communication between the upper and under side of the piston, as already described.

The pumping is performed wholly by the weight of the pump-rods, and these are generally much heavier than the column of water to be raised; and counterpoises are affixed to balance and sustain some part of their weight. It will be seen, from the above description of the working of the Cornish engine, that it differs from all the steam-engines already described: the steam is of a very high pressure, and it is employed in a different manner, the object being first to start the enormous mass of pump-rods, the weight and friction of which it is very difficult to estimate; and, having put this enormous mass in motion, to deal with it in such a manner that it may be available for pumping on the return stroke.

The difference between the two modes of applying steam, and the application of highly elastic and atmospheric steam, is so great, that theoretical calculations of the duty of the Cornish engine made on the same principles as the calculations of the duty of the atmospheric engine proved fallacious *.

190*a*. *Duty of Steam-engines.* The term *duty*, as introduced by Watt, may be defined to be the weight of water raised one foot high by a given quantity, as, a bushel of coals. When Watt applied his improved engine to the mines, he was to be paid a certain share of the saving in coals as the rent of his engine; hence the saving in coals over other engines formed a very convenient commercial criterion of the merit of the engine as between its maker and

* The student is referred for further information on the Cornish Engine to the papers by Mr Enys, Mr Henwood, Mr Parkes, and Mr Wicksteed, in the Second and Third Volumes of the *Transactions of the Institution of Civil Engineers*, 4to.

employer. Thus the work done by a bushel of coal (94 lbs.) became a measure of the duty done by a pumping engine; and engineers have been so accustomed to the standard, that it has been generally resorted to, although the quantity of steam used, or water as steam*, is obviously the only proper standard of comparison, inasmuch as the steam generated by the bushel of coal may be employed with very different efficiency in different classes of engines. The comparison between engines of the same class and having similar proportions of parts and boilers, and using the same kind of fuel, and worked under similar circumstances, may be well made by the quantity of fuel employed; but, where these elements are different, the usual standard of comparison is necessarily uncertain.

The duty being the number of pounds of water raised one foot high by a bushel of coals, will obviously be affected not only by the economy with which the steam is generated and applied, but by causes wholly unconnected with the engine, as the reduction of pit-work friction, that is, the friction of the pump and pump-rods and balance-weights, and the greater depths of the shafts, and the improved workmanship of all the parts of the machinery; so that the amount of duty has been gradually progressive from the time of Watt to the present time†.

* See paper by Mr Parkes, *Transactions of Civil Engineers*, Vol. III. 4to. pp. 3 and 69.

† Mr Enys gives the following interesting facts as shewing the progress of duty. In 1778 Newcomen's most improved engine raised 7,483,163 lbs. one foot high with 94 lbs. of coal, the evaporation of water being from 6 to 7 cubic feet for the same quantity of fuel; average duty of other engines under 5,000,000; in 1798 Watt's low-pressure engine, average of four best engines, 28,847,173; of 23 engines 17,770,000, evaporation of water from 9 to 10 cubic feet; in 1838 from 70 to 80 millions, evaporation from 10 to 14 cubic feet, the steam being expanded from 2 to 6 times its volume at full pressure. See *Trans. Civ. Eng.* 4to. Vol. III. p. 449. The duty has in some instances ranged from 90 to 125 millions. *Ibid.* Vol. II. p. 67.

The performance of other engines than pumping-engines is commonly referred to the weight in pounds raised one foot during a minute of time; that sum being divided by 33,000 gives the expression of effect in terms of *horse-power*, and by this means various descriptions of engines are brought to a common standard of comparison*.

The term *duty* is also applicable to locomotive engines. The work done being a load drawn whose resistance may be taken at a certain number of pounds per ton, the duty done is the same, whether the engine be employed in drawing a load of a resistance measured by weight, or in raising the same weight; hence, the resistance being ascertained, or assumed in pounds, for a given velocity, the duty will be found by multiplying that resistance by the whole distance travelled over, and dividing the result by the number of pounds of fuel or water consumed, supposing the weight of water or steam to be adopted as the standard.

191. *Other applications.* It will be impossible, within the limits of the present treatise, to point out all the various modes in which steam is applied, but the following may be mentioned in illustration of its properties.

Steam-Gun. The elastic force of steam at very high temperatures is such, that Mr Perkins has attempted to apply it to the purposes to which gunpowder is applied. Small bullets are admitted down a tube into the breech of a gun-barrel, and a small quantity of highly elastic steam enters and drives them out.

A great number of small bullets may be thrown in rapid succession and flattened against an iron target at the distance of 30 yards; but larger masses, as cannon-balls, do not acquire sufficient velocity for military operations.

* The horse-power is taken at 33,000 lbs. raised one foot high in a minute.

Nasmyth's Steam-hammer. One of the most ingenious and instructive applications of highly elastic steam, is that exhibited in the steam-hammer of Mr James Nasmyth. The mode of driving tilt-hammers for forging large masses of iron is well known; these hammers are subject to many objections, among which, the want of parallelism between the anvil and the face of the hammer is the most serious. By Mr Nasmyth's direct action steam-hammer, the hammer is raised vertically and falls vertically, so that the face of the hammer and anvil will always be parallel to each other, and the mass of metal will be struck directly and between two parallel faces. The hammer is connected with a piston moving in a cylinder, and highly elastic steam is admitted to the under side of the piston; the piston and its attached hammer are thereby raised to the required height or the extent of the stroke, and the steam being allowed to escape into the air, the hammer falls with a momentum due to its weight and the height from which it descends; the resistance from the friction of the piston being extremely small. The facility with which this hammer can be adapted to varying circumstances is remarkable; a great number of small taps with almost incredibly rapid succession may be given by letting the steam on and raising the piston a part of the stroke; if the force of the blow is required to be broken, the steam is let on just before the fall is complete, by which means a cushion of steam is presented, and by which the fall may be modified to any extent; if it be desirable to add to the force of the fall, the steam may be let on to the upper side of the piston, and impel the piston by its elastic force during the descent; or a steam or air-cushion may be employed to produce a recoil by the compression received during a portion of the ascent of the piston*.

* The controul under which a hammer weighing many tons may be kept is incredible; it seems to move like a cork. The author has seen

Nasmyth's Pile-driving Machine. The steam-hammer action as above explained, has also been applied by its ingenious author to driving piles by raising up and letting fall the hammer or monkey without the intervention of any rotative motion*.

191 *a.* In the preceding articles, we have endeavoured to give a general idea of the way in which the elastic fluid produced by heat from water may be employed as a moving power. The general principles are the generation of the steam, its application, and its reconversion to the liquid state. Its generation consists simply in exposing water to heat: its applications are various; in the atmospheric engine it is used as a means of creating a vacuum, so that the weight of the atmosphere may be available; in the single- and double-acting engines it is used to create a vacuum so that its own elastic force may be available: in the non-condensing engines no vacuum is created, but the elastic force of the steam is itself sufficient as a moving power. It would be impossible to insert here any account of the practical construction of the steam-engine—of the various kinds of boilers devised to generate the greatest quantity of steam with the least consumption of fuel, and at the least expense of wear—of the uses of the beam—of the fly-wheel for storing up power and regulating the motion of the machinery to be driven—of the various valves designed to be worked by the smallest expenditure of power. The principles of the application of steam are the same, whatever mechanical arrangements may be adopted.

a person holding a nut in the fingers of one hand, and the handle of the steam-valve in the other, and cracking the nut by the descent of the hammer.

* These ingenious applications of steam are the subject of letters patent, granted December 9, 1842, and July 22, 1843, to their author, Mr James Nasmyth, of Patricroft near Manchester. For a more deailed account of these inventions, see Holtzappfel's *Turning and Mechanical Manipulation*, Vol. II. pp. 959 and 961.

The steam-engine as it at present exists owes its perfection to the genius of Watt; and the successive improvements, suggested by difficulties which presented themselves, form a most interesting and instructive history.

There is one part of the engine which has not yet been mentioned, but which is most essential in the application of steam as a motive power; it is called the *Governor*. Uniformity of motion is the most important point to be attained in all machinery, and the speed with which the piston works depends in a great measure on the quantity of steam which is supplied to the piston in the cylinder. If then the engine goes too fast this quantity must be diminished, and if too slow it must be increased; and the engine is made by means of the governor to regulate this supply of itself, so that when the engine has once started it will continue to go at the same rate.

CHAPTER XIV.

ON EVAPORATION.

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192. We have seen in the last chapter that vapour is rapidly formed at the surfaces of fluids during ebullition; but vapour is also formed at the surfaces of fluids at nearly all temperatures, and without the appearances of ebullition. This imperceptible formation of vapour at low temperatures is termed *evaporation.*

The consideration of the chief circumstances which influence the process of evaporation, may lead to a true theory respecting it.

(1) Evaporation is entirely confined to the surface of fluids, and is therefore proportional to the extent of surface. Thus if the same quantity of water be spread at a shallow depth over a large extent of surface, it will evaporate much faster than if it be placed in a narrow deep vessel.

If the surface of water be covered with oil, the evaporation is stopped altogether.

(2) The rate of evaporation depends on the temperature. If equal quantities of water be placed in two equal vessels, one of which is placed in a warm, and the other in a cold situation, the former will be quite evaporated before the other is apparently diminished at all.

(3) When the air is dry, the evaporation is rapid even at low temperatures; but if the atmosphere contain much vapour, the evaporation is extremely slow, and will be entirely suspended when a certain quantity of vapour has accumulated over the surface of the water.

(4) The evaporation is far less in still air than in a current, which is owing to the vapour accumulating above the surface (3); but when there is a current the vapour is conveyed away nearly as fast as it is formed.

(5) Evaporation is most rapid *in vacuo:* the pressure of the air on the surface of a fluid positively retards the evaporation, as is shewn at once by placing ether under an air-pump, when, on exhausting the air, vapour rises so rapidly as to produce ebullition.

(6) Since a large quantity of heat passes from a sensible to an insensible state during the formation of vapour (Art. 179), it follows that cold must be produced by evaporation.

This is beautifully exemplified by Leslie's mode of producing ice by evaporation.

A small vessel of water is placed under the receiver of an air-pump, and the air being exhausted the evaporation proceeds rapidly, but the vapour collecting above the surface of the water, will put a stop to the evaporation unless it be removed.

If a small vessel of sulphuric acid be put under the receiver with the water, the vapour will be absorbed as soon as formed, and the cold thus produced by the evaporation will freeze the water in the middle of summer.

Liquids which evaporate more rapidly than water cause a still farther reduction of temperature; and the evaporation of ether under the vacuum of an air-pump has produced cold intense enough to freeze mercury.

193. Prop. *The actual quantity of vapour which can exist in any given space depends solely on the temperature.*

From the experiments of Dalton, it appears that if a little water be put into a dry glass flask, and a thermometer be placed in it, when the thermometer stands at 30^0, the

quantity of vapour formed will be very small. If the thermometer stand at 40^0, more vapour will exist in it; if at 60^0, still more, and so on. If when the thermometer is at 60^0, the temperature of the flask be suddenly reduced to 40^0, a portion of the vapour will instantly be reconverted into water, and the quantity which remains in the flask will be precisely the same as when the temperature was originally at 40^0. And the elastic force of the vapour is always the same at the same temperature.

But the important point in these experiments is, that the *result* is precisely the same whether the flask be full of air or perfectly empty. The only difference is in the *rate* of evaporation (Art. 192, 5), for the flask, if previously empty, becomes full of vapour almost immediately, whereas the existence of the air presents a mechanical impediment to the diffusion of the particles of the vapour throughout the flask, since these particles coming in contact with the particles of the air, experience an obstruction precisely similar to that which a stream of water experiences in descending among pebbles.

It follows therefore that the existence of the air has nothing to do with the quantity of vapour which exists in any space, but that this quantity depends solely on the temperature.

194. Prop. *The quantity of water evaporated at any temperature is proportional to the elastic force of the vapour due to that temperature.*

It appears (Art. 193), that the quantity of vapour formed insensibly by the process termed evaporation depends simply on the temperature; hence under similar circumstances the quantity of vapour formed is proportional to the temperature.

Now the elastic force of vapour depends also on the temperature; hence the quantity evaporated at any tem-

perature ought to be proportional to the elastic force of the vapour at that temperature; and this is found to be the case, supposing no vapour to exist in the air*.

The table of the elastic force of vapour as calculated by Dalton, from the temperatures $-40^0 F.$ to $212^0 F.$, at the end of the volume, shews that the elastic force increases at a greater rate than the temperature.

195. Prop. *When the evaporating force is greater than the elastic force of the vapour existing in the atmosphere, the evaporation will proceed, and, when equal, it will be suspended.*

The obstruction which evaporation experiences, and which sometimes causes it to be suspended altogether, cannot proceed from the weight or pressure of the atmosphere, for in that case there could be no evaporation at temperatures below $212^0 F.$; but it must proceed from the *vis inertia* of the particles of air which the particles of the vapour encounter; for we have seen (Art. 193), that the ultimate quantity of vapour formed, is the same in a flask, whether it be full or empty of air. The air then having nothing to do with the quantity of vapour formed, cannot be the cause which suspends its formation, but the cause must be sought for in the elastic force of the vapour already formed, and which is collected above the evaporating surface.

If this be the true cause, it follows, that when the evaporating force is greater than the elastic force of the vapour already existing, the evaporation will proceed, and when an equilibrium subsists between these two forces, it will be suspended.

Now the evaporating force *in vacuo* is equal to the elastic force of the vapour due to the temperature of the

* Dalton, *Manchester Memoirs*, Vol. v.

evaporating surface, for the space becomes instantly full of vapour of this elastic force, and then no more vapour is formed, or the evaporating force is in equilibrium with the elastic force of the vapour.

But in the atmosphere, where the rate of evaporation is impeded by the inertia of the particles, the aqueous vapour which exists will often be of less elastic force than what is due to the temperature of the evaporating surface, and then evaporation will proceed at different rates, according to the temperature of that surface, and the difference between the elastic force of vapour at that temperature and the elastic force of the vapour already existing in the atmosphere.

Thus it appears, that the air can never be absolutely free from vapour; for ice and snow evaporate at exceedingly low temperatures, and the less vapour there is existing in the atmosphere, the greater tendency is there to the formation of more.

But it must be observed, that these remarks tacitly assume the truth of Dalton's hypothesis respecting the action of the particles of elastic fluids (Art. 94); that hypothesis must stand or fall according as it does or does not serve to explain the phenomena which occur; and in the present instance it is needless to insist on the explanation which the preceding statements afford of the various known phenomena of evaporation (Art. 192).

196. Prop. *To determine the quantity of vapour which may be raised at any time by evaporation.*

Let V be the quantity of dry air, and p its elastic force; and let V' be the quantity when saturated with vapour, whose elastic force is f.

Now the elastic force of air varies inversely as the space it occupies. Hence the elastic force of the dry air in the mixture

$$: p :: \frac{1}{V'} : \frac{1}{V};$$

$$\therefore \text{ its elastic force} = \frac{Vp}{V'};$$

and the elastic force of any mixture is the sum of the elastic forces of the component fluids (Art. 95);

$$\therefore p = \frac{Vp}{V'} + f; \qquad \therefore V = \frac{V'(p-f)}{p};$$

whence the quantity of dry air in a given mixture V' is known.

$$\text{Also, } V' = \frac{Vp}{p-f}, \text{ and } V' - V = \frac{Vf}{p-f}.$$

But $V' - V$ is the quantity of vapour which can become diffused through a given volume V of dry air.

Generally, let x be the quantity of vapour which can exist in a given unit of bulk, then making $V = 1$, we have,

$$x = \frac{f}{p-f}.$$

By means of these formulæ and the tables furnished by Dalton, we can ascertain the actual quantity of vapour which may exist in the atmosphere at any given temperature, and also the quantity of vapour which may rise before the saturation is complete.

Suppose the temperature to be 60^0; the elastic force of vapour at this temperature $= \cdot 52$.

The quantity of vapour therefore which may exist, is

$$x = \frac{\cdot 52}{30 - \cdot 52} = \frac{\cdot 52}{29\cdot 48}$$

$= \frac{1}{56}$th of any given volume of atmospheric air.

But suppose that the elastic force of vapour already existing in the atmosphere is $\cdot 26$.

Then the amount of the evaporating force which will be effective in raising more vapour, may be considered as

$$= \cdot 52 - \cdot 26 = \cdot 26,$$

which equals the elastic force of vapour at temperature 40^0. So that although the temperature is 60^0, yet in consequence of the vapour already existing, no more will evaporate from a given surface than would evaporate at 40^0, if the atmosphere at that temperature were perfectly dry. Here,

$$x = \frac{\cdot 26}{30 - \cdot 26} = \frac{26}{2974} = \frac{1}{114};$$

or $\frac{1}{114}$th of any given volume is the quantity of vapour which can be raised.

197. *Hygrometers.* The quantity of vapour present in the atmosphere is very variable, and instruments, called hygrometers, have been invented for determining the quantity of vapour which exists at any time.

One kind of hygrometer is founded on the variation in volume and weight which some substances experience, according to the relative state of dryness or moistness of the atmosphere. Thus human hair elongates on imbibing moisture, and contracts again when dried; and a piece of glass weighs sensibly more from exposure to a moist atmosphere.

The hygrometer used by Dalton is a piece of whipcord about 6 yards long, fastened at one end and stretched over a pulley by a small weight: substances of this kind, in consequence of the twisting of their fibres, are shortened on imbibing moisture.

The rate of evaporation depends on the quantity of vapour in the atmosphere (Art. 196), and evaporation is always accompanied by the loss of sensible heat (Art. 179); hence, if the bulb of a thermometer be covered with a small piece of linen and moistened with water, the rate of

evaporation, as indicated by the fall of the thermometer, will shew the state of the atmosphere with respect to saturation. Such is Leslie's hygrometer.

Thirdly, when the air is saturated with vapour, if a body whose temperature is the least degree less be brought into it, dew is immediately formed on its surface by the condensed vapour. The degree indicated by the thermometer when dew begins to be deposited, is called the *dew point*. If the air is dry, the temperature of the body must be many degrees lower than the temperature of the atmosphere before dew will be deposited. Hence the difference between the temperature of the atmosphere and the dew point, indicates the state of the atmosphere as to saturation with vapour.

These are the principles on which hygrometers have generally been constructed.

198. *Clouds*. The process of condensation, by which the aqueous vapour, which is always suspended in the atmosphere, returns to the state of water, is directly the reverse of evaporation, and the vapour so suspended, and in the process of being condensed, presents those appearances which are called clouds.

We have seen (Art. 193) that the maintenance of the vapour in the atmosphere depends entirely on temperature; when then from any cause the temperature becomes lower than the temperature corresponding to the elastic force of the vapour so sustained, the vapour can no longer remain in the same state, but becomes partially condensed, and presents the same appearances as steam presents on first issuing into the cold air; thus partial condensation of aqueous vapour will be generally caused by an intermixture of two currents of air of a different temperature.

The suspension of clouds is an extremely difficult subject, and none of the various theories on this subject

seem yet to have met with a general reception; the different forms which they assume have been divided into seven modifications*.

Rain.

199. Prop. *Rain is the result of masses of air of unequal temperatures, and containing different quantities of aqueous vapour, being mingled together.*

This theory of rain having been advanced by Hutton, is very generally received; and the following argument may be adduced in its favour.

A volume of air of a given temperature can be charged only with a limited portion of aqueous vapour (Art. 193), and when the temperature of this air is diminished, it becomes capable of retaining less vapour, so that if it were not saturated before, it approaches rapidly to its point of saturation, and its temperature sinking lower than the temperature which corresponds to the quantity of vapour it contains, it must part with some of this vapour; but if the temperature of this portion of air should become raised by any means, it will be enabled to receive more vapour instead of being inclined to part with any.

Now when two volumes of air of unequal temperature mix together, since one will receive the heat which the other loses, so that the resulting temperature will be the mean of the two component temperatures, it might seem probable that the resulting capacity also for aqueous vapour would be the mean of the two, and thus there would be no disposition in the mixture to part with any vapour. But it appears (Art. 194), that the elastic force of the vapour necessary for saturation increases much faster than the temperature. Hence, when two saturated volumes intermix, the elastic force of the vapour will be greater than that

* See Howard's *Climate of London.*—*Enc. Metrop.* Art. *Meteorology.*

due to the temperature, and the air will consequently part with some of it.

This will be rendered evident by an example.

Let two volumes of air saturated with vapour at temperatures 40^0 and 60^0 be mixed together,

the elastic force of vapour at $40^0 = {\cdot}26$,
and ....$60^0 = {\cdot}52$;
the mean temperature will be 50^0,
and elastic force.. ....$\cdot 39$.

But the elastic force of vapour at temperature $50^0 = {\cdot}36$, there is therefore an excess of elastic force above that which is due to the temperature, which is $= {\cdot}03$. The whole vapour therefore cannot be retained, but some being parted with, will most probably descend to the earth as rain. But rain does not result always on the mixture of two masses of air, and on the preceding theory, ought only to be the result when the volumes of air are saturated with vapour.

For if air at 40^0 contains vapour whose elastic force $= {\cdot}211$,
and .... 60^0 .... $= {\cdot}313$,
the mixture at 50^0 will .... $= {\cdot}262$.

But the elastic force of vapour at $50^0 = {\cdot}36$; there is therefore an evident deficiency in the quantity of vapour which may be retained, and therefore such a mixture can part with no vapour, or no rain can descend.

This theory will account very generally for the observed phenomena respecting rain *.

The part of the Huttonian theory which is opposed to the present received theory is, that the vapour, being supposed chemically combined, is precipitated as rain in the chemical sense of the term precipitation; whereas the aqueous vapour existing according to Dalton's theory, as a

* *Enc. Metrop.* Art. *Meteorology.*

separate fluid diffused through the particles of the atmosphere, is, from the causes just mentioned, condensed into water, and so precipitated or caused to descend to the earth.

200. Prop. *The precipitated vapour will descend as rain, snow, or hail, according to the temperature.*

The combined theories of Hutton and Dalton will account for the return of aqueous vapour to the earth, either as rain, snow, or hail.

When (from the diminished capacity of the air for retaining the aqueous vapour in consequence of the diminution of temperature) a precipitation of that aqueous vapour takes place, a multitude of exceedingly small drops, collecting so as to form a cloud, will descend with a small uniform velocity. That this will be the case, may be inferred from the analogous instance of small spherical bodies, as shot, descending in water.

Thus it appears that clouds may descend very slowly; and this agrees with the observed fact.

If now during the descent of these drops they enter a quantity of air which is not saturated with vapour, and which (from the elevation of its temperature above the temperature which is due to the quantity of vapour that exists in it) is capable of imbibing vapour, these drops may return to a state of vapour, and the cloud will disappear. But if the precipitation continue, the lower strata of air having their full quantity of vapour, the small drops coalescing will form larger ones, and descend as rain.

If now during the descent of these small drops they encounter a mass of air whose temperature is below 32^0, they will be congealed and descend as snow, the small particles cohering and forming flakes instead of drops, in consequence of their not being fluid.

If the drops of rain formed by the coherence of the small particles of condensed vapour pass, during their descent,

through a mass of air whose temperature is less than 32^0, they may descend as hail.

Dew.

201. PROP. *Dew is the aqueous vapour of the atmosphere condensed on those surfaces whose temperature has been diminished in consequence of their excess of radiation.*

When vapour is condensed into small drops upon the surfaces of bodies on or near the ground, it is called *dew;* and the only difference betwixt dew and rain is, that the condensation of the vapour is in one case made at the surface of the body on which it is deposited, and in the other the drops are formed at some distance above the earth; the cause is the same in both cases, namely, cold operating on vapour.

The question is, How comes the diminution of temperature which gives rise to this condensation? For the answer to this question we are entirely indebted to Dr Wells, who in his Essay on Dew has set down a series of most accurate experiments, and deduced from them what appears to be a true theory of dew. The facts are briefly as follow.

If on a clear night one thermometer be placed on the grass and another a few inches above it, the thermometer on the grass will indicate a much lower temperature than the thermometer above it. If the sky become overcast, the thermometer on the grass instantly will rise. One case mentioned by Dr Wells is, that the temperature of a grass-plot at half-past nine was 32^0, that twenty minutes later, the sky having become overcast, it rose to 39^0, and when the clouds disappeared it again sunk to 32^0. Not only do clouds produce this remarkable effect, but the thinnest cambric handkerchief placed a few feet above the ground will produce a similar effect.

The connexion of these facts with the theory of Prevost (Art. 172) respecting radiation is obvious; there can scarcely

be any doubt that the effect produced on the thermometer, when any object is placed above it, is owing to the interchange of radiation. When the night is clear and nothing is placed above the thermometer, the surface of the earth, and of all substances on the earth, radiate freely into the atmosphere, and there being no objects which radiate back again, the temperature of these surfaces falls. But when the night is cloudy, or the surface is covered with anything, the interchange of heat prevents that rapid fall of temperature which must otherwise take place. If this be the true theory, there must be a difference of temperature in every body whose radiating power is different; that is, some substances must be much colder than others, and consequently, if dew be vapour condensed by the coldness of the surface, much more dew must be deposited on some surfaces than on others, and much more must be formed on some nights than on others.

Now what is the observed fact? all good radiators of heat, among which may be mentioned glass, the thread of the gossamer, wool, and leaves of plants, are covered with dew; whereas a polished metal plate will have scarcely any deposited upon it.

Again, dew is never abundant except in clear and serene weather, and never exists at all when the weather is both cloudy and windy. A very gentle breeze, from the assistance which it gives to evaporation, is on the whole favourable, but a windy night, from the equilibrium of temperature which it occasions by convection (Art. 169), prevents much dew from being deposited; and when this equilibrium is still farther brought about by the interchange of radiation from a cloudy sky, no dew whatever can be deposited; and these remarks are fully borne out by the observed facts.

Hoar Frost. The aqueous vapour having been deposited in small drops on the surfaces of those surfaces whose temperature has, by the excess of its radiation, fallen below

the temperature of the surrounding atmosphere, will be subject to all the variations of temperature which that surface experiences. Hence if, from a cloud coming over, the temperature of the surface rise, the dew will return to aqueous vapour; if on the contrary the temperature continue to fall, the temperature of the small particles of dew will fall with it, and finally be congealed so as to present the appearances known by the name of hoar frost. Thus in an open field every blade of grass will frequently be covered with the beautiful crystals of hoar frost, while the grass under the branches of a spreading tree, where the temperature has been kept up by the interchange of radiation, will be covered only with a small quantity of dew.

202. *On the quantity of rain, and origin of springs**. The vapour which is raised by evaporation returns to the earth either as rain or dew. Now the quantity of rain and dew which falls in England and Wales in the course of a year is variously estimated; the quantity of rain being calculated by Dalton at about 31 inches of water, and the quantity of dew at about 5 inches, thus making a return to the earth of about 36 inches in the course of the year; and this would appear to be an approximation applicable to many places; but the quantity varies very much for different localities.

From a series of observations extending over fifteen years, by Mr H. H. Watson, of the amount of rain and evaporation at Bolton-le-Moors, in Lancashire, the greatest quantity of rain was 62 inches (in 1831), and the least $34\frac{1}{2}$ (in 1844), the mean or average quantity being about 47 inches.

According to observations on the Penine chain of hills, nearly along the line of the Sheffield and Manchester Rail-

* Dalton, *Manchester Memoirs*, Vol. v.

way near Glossop, the rain which fell at the foot of the hills was 45 inches on the average of 2½ years, and at different parts of the summit, about 1000 feet above the base, 67 and 77 inches*. It should be observed with reference to these observations, that the summit upon which they were made presents the first high ground to the masses of vapour borne from the St George's Channel by a north-west wind, the prevailing wind in that quarter. Other observations made on high ground near the Bann Reservoirs, in the north of Ireland, by Mr Bateman, shew the great excess of rain which is deposited on high lands, in localities situated to catch the vapour as it is brought from the ocean by the prevailing wind.

The mean or average of 3 years' rain near the west coast of Scotland at Gilmourton Avondale, at the level of 600 feet above the sea, was 49·3 inches, while the mean of 16 years near the east coast, at Glencorse in the Pentland Hills, at the level of 734 feet above the sea, was 25·6 inches †.

According to observations extending over nine years in the English Lake District, by J. F. Miller, of Whitehaven, it appears that at Seathwaite, the centre of that district, the greatest fall of rain in any one year was 160·9 inches, in 1847; and the least, 113 inches, in 1853. Local causes materially affect the quantity of rain falling at different spots, but the level of the most heavily charged rain-cloud and of the greatest fall of rain, would appear to be in that district at about 2000 feet above the level of the sea ‡.

The fall of rain near the east coast of England at Boston, at the level of 40 feet above the sea, was 22·3 inches as the mean of 17 years, thus shewing a remarkable contrast between

* See Paper by J. F. Bateman, read before the Literary and Philosophical Society of Manchester, February 6, 1844.

† See N. Beardmore's *Hydraulic Tables*, p. 33.

‡ See papers by J. F. Miller in the *Transactions of the Royal Society of London*, 1850, 1851, and *of Edinburgh*, 1855, and in *Edinburgh Philosophical Journal*, April, 1854.

the rain-fall on the east and west coasts, and the effect of high land in intercepting the clouds of vapour from the Atlantic impelled by westerly winds *.

Under ordinary circumstances, the quantity of rain collected by a rain-gauge at the surface of the earth, is greater than at more elevated situations †; the drops of rain once formed accumulate moisture by condensation at their surfaces during their descent, but upon ground so situate as to catch and break the clouds as they come from the ocean, the quantity of rain is much greater than that which would fall on the lower ground in the same locality.

The rain and dew may be regarded as equivalent to the quantity of water raised by evaporation and carried off by the rivers and by underground drainage. The water which sinks deep into some soils, gives rise to springs, some of which become languid towards the end of the summer, and cease to flow altogether after very long droughts.

Of the 36 inches of rain, it is supposed by Dalton, that 13 are carried off by rivers to the sea, and the remaining 23 raised by evaporation from the surface of the earth ‡; and there is every reason to conclude that the evaporation from the land and sea, and the return again of the vapour so raised to the surface of the earth, are kept up by the same general causes with great uniformity from year to year.

* At the British Association at Belfast, in 1852, Prof. Lloyd exhibited a table of the mean yearly rain in Ireland exhibiting the same class of phenomena, being on the west coast, at Cahireiveen, 59·4, and at Westport 45·9 inches, but on the east coast at Dublin, 26·4, and at Donaghadee, 27·9 inches.

† See *Transactions of the British Association*, 1834, for observations by Prof. Phillips, at York, whence it appears that the fall of rain during twelve months being 25·7 inches at the surface of the ground, was 19·8 inches at 44 feet, and 15 inches at 213 feet above the ground.

‡ The observations of Mr H. H. Watson give the mean of the evaporation during 15 years as 23 inches, the mean of the rain being 47 inches.

CHAPTER XV.

ON WINDS.

203. If an equilibrium of temperature prevailed all over the globe, the atmosphere would be in perfect equilibrium, its density and elastic force would be equal at equal elevations, and diminish as we ascend in a geometric progression.

But the temperature in the middle latitudes of the earth, modified as it is by an inconceivable variety of local circumstances, is extremely variable. If we consider the different positions of the equatorial and polar regions with reference to the great source of heat, the sun, and the different physical characters of these regions, it will be evident that their mean temperature must differ exceedingly. Now, from observations made near the equator, it is inferred that the mean temperature is 84^0 *F.*

The mean temperature of the polar regions is of course less than 32^0; and from the observations of Scoresby, it appears that the mean temperature of the parallels of $76^0\ 45'$ and 78^0 north latitude, is 18^0 and 16^0, whence it is inferred that the mean temperature of the pole is about 4^0 *.

Thus there is evidently a prodigious diminution in the mean temperature as we rise in latitude. Now the effect of increased temperature on a mass of air is that it increases in bulk, and becoming lighter than the surrounding air, rises up, and its place is supplied by a quantity of colder air. Hence if the atmosphere were in equilibrium, an increase of temperature at any one point would immediately disturb

* *Enc. Metrop.* Art. *Meteorology.*

that equilibrium, and motion would ensue; so that, independently of all other circumstances, the difference of temperature at the equator and poles would effectually disturb the equilibrium of our atmosphere, and establish two grand currents, an upper current towards the poles, and an under current towards the equator.

204. *Constant Winds.* But the effect of the gradual change in the temperature from the equator to the poles, in disturbing the equilibrium of the atmosphere, must be small when compared with the great and constant excess of temperature in the tropical regions of the earth, that is, for about 30^0 on each side of the equator. The uniform high temperature of these regions causes a constant and rapid transfer of the air from the lower to the upper parts of the atmosphere, and the colder air of the higher latitudes moves down into its place. On each side of the equator, then, there is an upper current moving towards the poles, which, if there were no other causes in operation, would travel due north and due south, that is, would have the character of a southerly and northerly wind in the north and south hemispheres respectively; and there is a lower current moving towards the equator, which, if there were no other causes in operation, would travel due south and due north, that is, to a person in the north tropics there would be a constant northerly wind, and to a person in the south tropics a constant southerly wind. But the directions of these currents are modified by the rotation of the earth, the upper one acquiring somewhat of a westerly, and the lower of an easterly character.

The quantity of air which passes below from higher latitudes to the equator must be exactly balanced by the quantity which flows above in the opposite direction; and this is known by observation to be the fact, for the mean height of the barometer is the same at the equator as in other latitudes.

205. *Variable Winds.* Every part of the earth's surface is, from local as well as general causes, subject to great variations in temperature, and the superincumbent atmosphere must participate in these variations. Hence since like effects will always follow like causes, there is no difficulty in accounting for the origin of innumerable different atmospheric currents such as are exhibited in the variable winds, although it may be impossible to foretell their directions, modified as they must all be by the two grand and predominant currents which exist in the upper and lower regions, and by the earth's rotation.

Hurricanes, &c. It seems worth enquiry (says Herschel*), whether hurricanes in tropical climates may not arise from portions of the upper currents prematurely diverted downwards before their relative velocity has been sufficiently reduced by friction on, and gradually mixing with, the lower strata, and so dashing upon the earth with that tremendous velocity which gives them their destructive character, and of which hardly any rational account has yet been given. Their course, generally speaking, is in opposition to the trade-wind (Art. 206), as it ought to be in conformity with this idea. But it by no means follows that this must always be the case. In general a rapid transfer either way in latitude of any mass of air, which local or temporary causes might carry above the immediate reach of the friction of the earth's surface, would give a fearful exaggeration to its velocity. Wherever such a mass should strike the earth, a hurricane might arise; and should any two such masses encounter in mid-air, a tornado of any degree of intensity on record might easily result from their combination.

The Trade-Winds.

206. Prop. *If the earth revolve from west to east, there must exist in the north tropical regions a constant north-*

* *Astronomy*, p. 132, note.

easterly, and in the south tropical a constant south-easterly wind.

The lower atmospheric current which exists in each hemisphere, in consequence of the excess of the temperature of the atmosphere in the tropical regions of the earth, must give rise to constant currents or winds of a north-easterly and south-easterly character, in the north and south tropics respectively.

If the earth were at rest these currents would be due north and due south winds, in the north and south hemisphere respectively (Art. 204).

Now if these currents did not exist, but the atmosphere were in perfect equilibrium, the earth revolving from west to east would either carry the atmosphere with it, in which case there would be a perfect calm; or it would revolve within the atmosphere, in which case there would be a perpetual current from east to west, or a constant east wind.

Again, since the diameters of the circles of the earth parallel to the equator increase from the pole to the equator, the velocity of any point on the earth's surface increases as we approach the equator. Thus the linear velocity of the motion of any mass of air must be much greater near the equator than at higher latitudes, and infinitely greater than at the pole, where it is nothing.

Now it is evident that the atmosphere, situated as it is with respect to the earth, must revolve round with it, so that if there were no disturbing causes there would be a perfect calm. But the lower currents as they move towards the equator do not acquire all at once the velocity of that part of the earth over which they are advancing; hence the earth in these parts may be considered as revolving within its atmosphere, and consequently an easterly current is created. Thus at a point within the tropics the atmosphere has the character of a constant easterly wind; but it has

also the character of a constant north and south wind, and therefore the resulting wind is north-east in one case, and south-east in the other; that is, there must exist a constant north-easterly wind in the north tropics, and a constant south-easterly in the south tropics.

Now as the lower currents approach the equator they acquire more and more the motion of the earth. Hence their easterly character is lost, and consequently just at the equator the northerly and southerly currents meet in opposite directions, and are therefore in equilibrium with each other, and thus the winds at the equator will be due to local causes simply.

The result then must be the production of two great tropical belts of atmosphere, in the northern of which a constant north-easterly, and in the southern a south-easterly wind must prevail, while the wind in the equatorial belt which separates the two former must be comparatively calm and free from the easterly character.

These consequences accord exactly with the observed fact*, and constitute the *regular trade-winds.*

207. PROP. *A constant south-west wind must prevail in the northern hemisphere, and a north-west in the southern, and a westerly wind in the extra-tropical regions.*

The atmosphere about the equator moves with a greater velocity than the atmosphere in middle latitudes; hence, when it rises up and creates the southerly and northerly upper currents, it will retain this velocity; and when, on having its temperature reduced, it returns again towards the surface of the earth, this velocity will be in no wise abated; and consequently the upper currents in our latitudes must move faster than the under; that is, they must have

* See Captain Hall's *Fragments of Voyages and Travels,* Second Series, Vol. I. p. 162.

more of a westerly character. Thus this upper current has a westerly tendency, and the resulting wind will therefore be south-westerly.

In the southern hemisphere the northerly upper current advances faster towards the east than the lower currents, hence the resulting wind is a north-westerly one.

These currents will be so modified by the currents created from local causes, that their direction at the surface of the earth will not always be south and north-westerly. But the origin of the south-westerly and westerly winds, so prevalent in our climate, and of the almost universal westerly winds in the extra-tropical regions, is thus clearly accounted for.

One other consideration is worth mentioning, which is, that these gales have the character of compensation currents for preserving the rotation of the earth. The constant north-easterly and south-easterly winds in the tropical regions must tend, by their constant action on the earth, being in a direction opposite to its motion, to destroy that motion; whereas these westerly, south-westerly, and north-westerly currents, by acting on the surface of the earth in the direction of its motion, must tend to accelerate that motion. Thus beautifully is the uniformity of the motion of the earth preserved by the perfect equilibrium which exists among the disturbing forces.

208. *Observed Phenomena.* The preceding is the theoretical conclusion deduced from general principles; and, though the trade-winds are great evidence of the truth of the theory, the confirmation of it for the upper current also is equally decisive. A volcanic eruption in the island of St Vincent, in 1812, supplied the necessary evidence *.

The island of Barbadoes is considerably to the east of St Vincent, and the trade-wind blows strongly betwixt them; but notwithstanding this, during the eruption at St Vincent

* *Blackwood's Magazine*, Vol. I. p. 134.

dense clouds were formed at a great height in the atmosphere above Barbadoes, and vast quantities of heavy dust fell on the island, and to a great distance out at sea, to the north-east of the island. Here then was an instance of what ought to be expected on this theory; the dust was by the volcanic eruption carried into the upper current, which, being westerly, carried it across to Barbadoes, in a direction exactly opposite to the easterly part of the trade-wind.

209. *Sea and Land-Breezes.* It is observed that generally during the day the wind sets from sea to land, and that during the night it sets from land to sea.

Now during the day the earth and the superincumbent atmosphere have a temperature which is above the mean temperature of the season, and consequently, the air becoming rarefied, rises up and is replaced by the cooler air from the sea, thus establishing a current which is called the *sea-breeze.*

During the night the temperature of the land and of the superincumbent atmosphere is somewhat below the mean temperature of the season, and therefore the air over the sea being somewhat warmer (since the sea preserves a temperature nearly uniform), rises up, and a current from the land is established, which is called the *land-breeze.*

210. Prop. *A large tract of continent will change the direction of a wind.*

This effect is seen in the land and sea-breezes, which is sufficient to change the direction of the prevailing wind. The western coasts of Africa and America, about the equator, become so heated by the action of a vertical sun, that the trade-winds lose their easterly character, and the prevailing wind at these parts is a westerly wind: a sea-breeze constantly sets in on these parts of the coasts.

Monsoons, and other periodical winds, admit of the same explanation. A particular distribution of land and water, acted upon by the periodical sun, produces periodical winds. When the sun is vertical the breezes are invariably sea-breezes, and when the sun declines the land cools much faster, and sinks to a much lower temperature than the surrounding water, and consequently the breezes become land-breezes, or their direction is entirely changed.

These remarks will be sufficient to afford some insight into the causes of the constant and periodical currents which constitute winds, and shew distinctly that a change in temperature, combined with the rotation of the earth, is their real cause; and the state of the weather at partioular places may be explained on the same principles*.

210 *a*. *Hurricanes, Typhoons, Rotary Storms.* The air in hurricanes and rotary storms has a motion of translation from the equator towards the poles, and of rotation about a centre in a direction *against* the sun in both hemispheres; that is to say, in the northern hemisphere the motion of the air in the rotary storm is from west by south to east, and in the southern hemisphere from west by north to east, or considering the motion of the hands of a watch as illustrating the rotation of the air, the northern storms rotate opposite to the southern storms with the direction of motion of the hands.

These phenomena are consistent with what the laws of fluids lead us to expect as resulting from the sudden transference of a mass of air upwards or downwards as in the analogous case of the vortices produced by the discharge of a liquid at a point below the surface but which continue after such discharge has ceased †.

* See Daniells' *Meteorological Essays.*

† See *Law of Storms* by Lieutenant-Colonel Reid.

CHAPTER XVI.

ON CAPILLARY ATTRACTION.

211. THE laws of the equilibrium of fluids, treated of in the preceding chapters, are subject to some remarkable exceptions when the spaces in which the fluid is contained are those commonly called capillary spaces. These phenomena, first observed in tubes of small internal diameter, present themselves in a greater or less degree in all tubes, or whenever a liquid is in contact with a solid. The general law arrived at (Art. 33) was, that when a fluid is in equilibrium every point on the surface is on the same level; but in these phenomena we know that the liquid is sometimes raised above, at other times depressed below, the surrounding liquid. Thus, if a tube of small internal diameter be inserted in water, the water in the tube will stand above the level of the water without the tube; if the tube be inserted in mercury the surface of the mercury in the tube will be below the surface of the mercury without the tube; and a liquid is drawn up or depressed by the side of any solid with which it is in contact. Again, two bodies near each other and floating on a fluid are sometimes attracted towards each other, and sometimes repelled. We have to investigate the laws of these phenomena.

212. *Forces of Capillarity.* By the term capillarity we express the peculiar forces or combined actions to which the phenomena in question are due. The forces to which

a liquid is subject are, the mutual attraction betwixt its own particles, or cohesion, the mutual attraction betwixt its particles and the particles of the solid, or adhesion, and the impressed force of gravity. The effect of the latter, and the laws of the equilibrium of fluids subject to it, have been already considered: we have now to consider the results of the modifying forces of cohesion and adhesion, as exhibited under the peculiar circumstances of a liquid contained in narrow spaces, or in contact with a solid.

The existence of the force of cohesion in a liquid is proved by the form which a drop of water or mercury will assume; by the tendency which it has to resume its form when slightly disturbed: the particles of every drop are owing to the action of these forces in a position of stable equilibrium. For we may conceive any horizontal section of a drop; all the liquid particles above this section must attract all those below it, so as to counterbalance the action of gravity upon them.

The existence of the force of adhesion is clear from the suspension of a drop of liquid from the under side of any body; indeed, this phenomenon proves the existence of the forces both of adhesion and of cohesion: it is the action of these, combined with the action of gravity, to which the form of the drop is due.

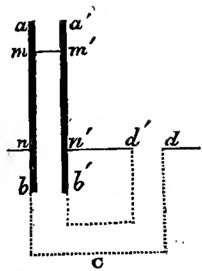
The amount of these two attractions may be measured by suspending a flat plate from the end of a scale-beam, and observing what weight at the other end will be requisite to raise the plate when in contact with the liquid. In general, the force of adhesion is much greater than the force of cohesion, and the plate when raised carries with it a film of the particles of the liquid. Plates of different substances, but equal area, require the same force to raise them from the same liquid, but every liquid has a force of cohesion peculiar to itself. When the plate does

not bring away any of the particles of the liquid, or, in other words, is not wetted by the liquid, the weight measures the force of adhesion betwixt the solid and liquid. This is found to vary with the nature of the plate and liquid, and with the time during which their surfaces have been in contact.

All the phenomena of capillarity are exhibited equally both in air and in vacuo, and are entirely independent of the thickness of the material composing the body in contact with the liquid. Thus, in glass tubes of equal internal diameter the liquid has the same elevation or the same depression whatever the external diameters of the tubes. Also, the elevation or depression is not proportional to the specific gravity of the liquid; water stands much higher in a glass tube than alcohol. It follows, then, that the phenomena must depend entirely on the mutual action betwixt the particles of the liquid and the attraction of the tube. Moreover, the thickness of the tube having no effect on the amount of elevation or depression, it follows, that the envelopes of matter situated at an appreciable distance from the interior of the tube can have no effect on the height of the column. We may then conclude that the phenomena of capillary spaces are due simply to the molecular attraction subsisting betwixt the particles of the liquid and to the action of the particles of the solid body on the particles of the liquid, which latter action is insensible at sensible distances. The absolute intensity of these forces of cohesion and adhesion are unknown, but we can from capillary phenomena form some estimate of their relative value.

213. Prop. *To explain generally the elevation and depression of the liquid in a capillary space.*

Let mm' be the surface of the column mn of a liquid suspended in a capillary space $abb'a'$ above the surface



$n\,n'$ of the external liquid. The equilibrium being established, any line of liquid particles may be taken and supposed to be detached from the rest, and inclosed in a tube, without altering the forces exerted (Art. 30). Let the line of particles included betwixt the dotted lines be conceived so detached.

The actions which the particles of the liquid included in the tube exert on each other, or sustain from the sides of the tube, have no tendency to make the liquid move either up or down. But the column $m\,b$ has some action exerted upon it by the sides of the tube situated above the surface $m\,m'$. Let F represent this upward action due to the tube. The column is also attracted downwards by the detached column $b\,c$, that is, by the liquid contained in the imaginary tube. Let F' represent this downward action of the liquid. Also the part $b\,c$ of the liquid is attracted upwards by the tube $a\,b$, and F will represent this action.

Thus the liquid column is acted on by two upward actions, which we may represent by $2F$, and by a downward action, represented by F'. The whole force causing the ascent is therefore $(2F-F')$, and this must be in equilibrium with the weight of the sustained column.

Let h be the height of the elevated column, a its section, and ρ the density of the liquid; the weight then will be $g\,\rho\,a\,h$, and we consequently must have

$$2F-F'=g\,\rho\,a\,h.$$

Let the tube be cylindrical, and r be its radius. Then since it appears from experiment that the energy of the force of ascent depends in some manner on the diameter of the column, we may assume

$$F=2\,\pi\,r\,\alpha, \quad \text{and } F'=2\,\pi\,r\,\alpha',$$

where a, a', are quantities independent of the form and thickness of the tube, and express specifically the action betwixt the particles of the liquid and of the solid on the liquid. Also $a = \pi r^2$. Substituting these we have

$$2\pi(2a - a') = \pi r^2 g\rho h, \quad \text{or } h = 2\frac{2a - a'}{g\rho}\frac{1}{r}.$$

The value of h will depend on the quantity $2a - a'$; it will be positive when $2a$ is $> a'$, nothing when $2a = a'$, and negative when $2a < a'$; in this latter case the column h will be depressed and not elevated.

Thus the relative intensities of the actions of cohesion and of adhesion determine the phenomenon of ascent or descent.

For the same liquid and tube the quantity $\dfrac{2a - a'}{g\rho}$ is invariable; consequently the elevation or depression of a liquid in tubes of a given material is inversely as the radius or diameter of the tube.

We shall see presently (Art. 217) a more exact method of arriving at the preceding conclusion.

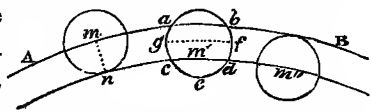
214. *Laplace's Theory.* There is another mode of viewing capillary phenomena, which enables us to explain in a more complete manner the different modifications which they may present. Whether the liquid column within the capillary tube, which we have been speaking of, is raised or depressed, its upper surface cannot be a plane; for if we imagine it to be composed of concentric annular portions, none of them but that which touches the side of the tube will be directly affected by the capillary action; the others will rise by the attraction which takes place between the particles of the liquid itself: it is evident from this that the upper surface must be concave or convex, according to circumstances. There is then a necessary connexion between the form of the unconfined surface of the fluid column within the

capillary cavity, and its state of elevation or depression. Now Laplace has shewn that this connexion may be established directly, without any reference to the action of the solid matter on the liquid.

In order to present the important principles involved in this question as clearly as possible, we will consider in the next article the action on any particle at the surface, when the liquid is not contained in a capillary space; but when the surface is free, or a level surface.

215. Prop. *The particles at the surface of a liquid are subject to an action resulting from the attraction of the particles in the interior of the liquid.*

The molecular attraction of the particles of a liquid extends only to a small distance; the distance through which this extends may be said to be the sphere of the sensible attraction or activity of the particle.



Let AB be the surface of any liquid at rest, and from any point m as a centre, let a sphere be described with a radius equal to the greatest distance at which the attraction of the particles on themselves is sensible. The particle at m will only be attracted by the particles within this sphere, and the resultant of these attractions will evidently be in the direction of the normal mn. Let any other point as m' be taken within the liquid; let a sphere be described as before, and through m' draw the line gf parallel to the surface; draw a line cd parallel to the surface at a distance from it equal to the radius of this sphere of attraction. Then the resultant of the attraction of the particles of the liquid comprised betwixt gf and cd and betwixt gf and ab on the point m' are in equilibrium with each other, and this point is attracted only by the particles in the segment ced. For a particle m'', situated

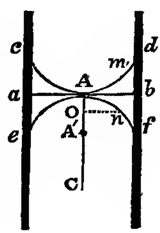
at a distance below the surface equal to that of sensible attraction, the sphere falls entirely within the liquid; the particle is equally attracted on all sides, and consequently in equilibrium: the same also is the case with all the particles situated at a greater depth within the interior of the liquid. It follows, therefore, that all the particles contained betwixt the surface of a liquid and an interior surface parallel to the former, are attracted in the direction of the normal to the surface; that this attraction is greatest at the surface, and diminishes for the particles within the surface, and becomes finally insensible.

The resultant of all these forces on a particle situated at the plane surface of a liquid will in the following articles be called A.

When the liquid is contained in a capillary space, so that the surface is convex or concave, the action on a particle at the surface will be greater or less than in the preceding case, as we shall see immediately.

216. Prop. *To determine the nature of the force on a particle situated at a point of curve surface in a capillary space.*

The pressure which we have called A (Art. 215) occurring in the mass of a liquid whose surface is horizontal or plane, is diminished when this surface is concave towards the interior, and increased when convex. The cause of this change will be understood from the following reasoning.



Let cd, ef, represent the concave and convex surfaces of a liquid in a capillary tube. The action of the particles of the fluid in the meniscus $caAbd$, on the central filet AC, evidently has a resultant directed upward in the direction CA, whose magnitude we will call M; so that the definite pressure A which the molecules of the line AC situated at a

depth greater than the radius of the sphere of molecular attraction would sustain if the surface were plane (Art. 213), is only equal to $A - M$ when the surface is concave.

To determine the value of the pressure when the surface is convex, we must take from A the action of the meniscus $eabf$, which we will suppose of the same form as the other. It will be evident that the action of this meniscus on the central line must be exactly equal to the action of the former one.

Let us consider any particle n of this new meniscus. Draw nO perpendicular to AC and take $OA' = OA$; the action of n on the particles of the liquid line comprised betwixt A and O have a tendency to make it descend; the action of n on the particles of the filet betwixt O and A' tend to make it ascend; these two contrary efforts being equal will destroy each other; so that n will act with effect only on the particles situated below A'. Its action then will be equal to that of a molecule m of the first meniscus, symmetrical with n as regards the plane aAb. The same identity of action will take place for all the molecules symmetrically placed in each meniscus. Each meniscus then acts in the same manner on the filet AC. But the action of the meniscus $cabd$ being M, that of $eabf$ will be M also, acting upwards; hence, since we may treat the column terminated by the surface eAf as a column terminated by a plane surface minus the meniscus $eabf$, it follows, deducting the upward force due to this, that the pressure exerted along AC, the surface being convex, is $A + M$. Now the value of M being calculated by analysis we have*,

$$M = \tfrac{1}{2}\left\{\frac{1}{R} + \frac{1}{r}\right\}B,$$

where R and r are the radii of greatest and least curvature of the surface at the point A; and B is the value of an integral.

* See *Theory of Fluids*, Art. 88.

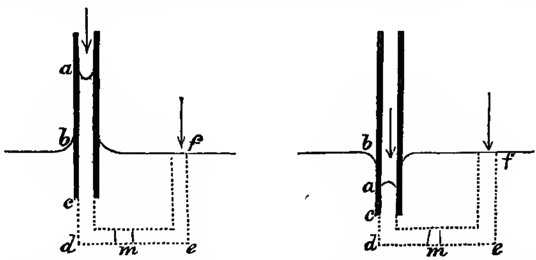
The pressures therefore exerted on a point of a surface concave or convex are respectively,

$$A - \tfrac{1}{2}B\left(\frac{1}{R}+\frac{1}{r}\right), \qquad A + \tfrac{1}{2}B\left(\frac{1}{R}+\frac{1}{r}\right).$$

The preceding theoretical results are confirmed by experiment; for we observe that a solid tube being immersed in any liquid, the level of the liquid in the interior of the tube stands above that of the external liquid, whenever the surface is concave, is the same when the surface is plane, and depressed below when the surface is convex. Thus the form of the surface and the elevation of the column are connected by an invariable law.

217. Prop. *To determine the elevation or depression in a capillary space.*

Let the accompanying diagrams represent a capillary tube inserted in a liquid; in one of which the column



is elevated, in the other depressed. We have to consider the forces to which this column is subject. Let any line of particles whereof *de* is parallel to the free surface of the fluid, that is, perpendicular to gravity, be conceived detached. Then since there is equilibrium the pressures must be equal in each direction on any particle at *m*. The pressure transmitted to *m* along the line *a b d* is that due to the weight of the column *a d*, and the action of the liquid on the particles of the curved

surface; the action transmitted along fe is that due to the weight of the column fe, and the action of the liquid on the particles of a plane surface. Consequently, if h be the height of the column above the surface, and z the depth of m below the surface, we have

$$g\rho(h+z)+A-\tfrac{1}{2}B\left(\frac{1}{R}+\frac{1}{r}\right)=A+g\rho z;$$

$$\therefore g\rho h=\tfrac{1}{2}B\left(\frac{1}{R}+\frac{1}{r}\right).$$

When the column is depressed we have similarly

$$g\rho(z-h)+A+\tfrac{1}{2}B\left(\frac{1}{R}+\frac{1}{r}\right)=A+g\rho z;$$

$$\therefore g\rho h=\tfrac{1}{2}B\left(\frac{1}{R}+\frac{1}{r}\right).$$

Thus the amount of elevation and depression is expressed in terms of the radii of curvature of the surface, and each is inseparably connected with the peculiar form of the surface as to concavity or convexity.

The tube being cylindrical the liquid will be terminated by a surface of revolution, and $R=r$; consequently

$$p=\frac{B}{g\rho}\frac{1}{r};$$

that is, the elevation or depression is inversely as the radius of curvature of the surface at the lowest point. In this case the capillary surface will be very nearly spherical; and the elevation or depression inversely as the radius of the tube*.

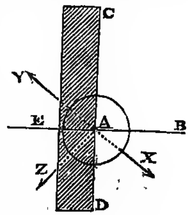
218. PROP. *The surface of a liquid in contact with a solid will be plane, concave, convex, according as the attraction of the particles of the liquid for each other is equal*

* *Theory of Fluids*, Art. 89.

to, less, or greater than, twice the action of the solid on the liquid.

The reasoning in preceding propositions depends on the form of the surface of the liquids; as its concavity or convexity: it remains to shew on what each of these depends. At first sight it would appear as if a body would be wetted by a liquid when the attraction of the solid for the liquid is greater than of the particles of the liquid for each to her.

Let AB be the surface of a liquid in contact with a solid whose face is CD. The action of the liquid betwixt AB and AD will have a resultant bisecting the angle DAB as represented at X; the resultants of the actions of the particles of the solid above and below the surface of the liquid will bisect the other angles, as represented by Y and Z. The action then on any particle situated at A will be the resultant of these three forces, and the direction of this resultant will determine the form of the surface of the liquid at the point A; for the surface must be perpendicular to this resultant.



If this resultant be parallel to AD, that is, perpendicular to the surface AB, the surface of the liquid will be plane; if the resultant lie in the angle BAD, the surface will be convex, and if in the angle CAE, it will be concave.

To determine the cases in which this will occur, let P represent the action of the liquid in the direction AX, and Q the action of the solid in the directions AY and AZ. The forces Y and Z being resolved into the horizontal and vertical direction, the two latter will destroy each other, and the sum of the former will be $2\,Q \cos 45^0$. Resolving X in the same manner, each portion will be $P \cos 45^0$; the horizontal part of which will be opposed to the horizontal

portion of the former. Hence, the total force on a particle at A in the horizontal direction

$$= (2Q - P) \cos. 45^0,$$

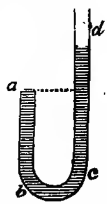
the total force in the direction $AD = P \cos. 45^0$.

That the surface at A may be plane, the only resultant force must act in the direction AD, which is impossible, unless $2Q - P = O$.

That the surface may be convex, the resultant must be in the angle DAB; the horizontal force must consequently be in the direction AB, or must be negative, that is, $2Q$ must be $> P$.

That the surface may be concave, $2Q$ must be $< P$. Thus the surface will be plane, convex, or concave, according as the cohesion is equal to, greater, or less than twice the adhesion. The same result was obtained before (Art. 216).

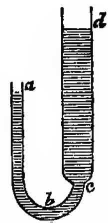
219. *Experiments.* The preceding reasoning and conclusions are verified in a remarkable manner by the following experiments, which shew distinctly the influence of the curvature of the bounding surface of a liquid contained in a capillary tube. Let a liquid which will wet the sides be poured into a capillary tube bent as represented in the figure, so that one branch is shorter than the other.



The two branches having the same diameter the liquid will stand at the same height in both. But if liquid be poured into the shorter leg $a\,b$ so as nearly to fill it, the concavity of the surface will diminish, the surface will at one instant be apparently plane, and on adding more liquid it will stand above the edge of the tube, and become convex. During these changes of form the liquid in the branch $d\,c$ will raise itself more and more above the extremity a: the surface being plane, the difference of level will be equal to that which would exist on the tube being inserted into the

liquid; and this becomes doubled when the surface of the liquid at a is convex. This result is in accordance with the preceding theory. For the pressures being $A-M$ and A, their difference is M, and in the second being $A-M$ and $A+M$, their difference is $2M$; the elevation is consequently double in the latter case.

The following experiment also will verify the preceding reasoning. Let a capillary tube $a\,b$ perfectly dry be connected with a tube $c\,d$ of sensible diameter; a liquid poured into $c\,d$ will rise in $a\,b$, and be terminated by a concave surface. If now the tube $a\,b$ be not wetted above the surface of the liquid, and if additional liquid be poured gently into the tube $c\,d$; the curvature of the liquid in $a\,b$ will gradually diminish, and become plane, and finally convex. The difference of level at the same time will experience corresponding variations; it will diminish with the curvature, and vanish when the surface is plane, and there will be a depression when the surface is convex.



The elevation or depression of a liquid betwixt two glass plates is inversely as the distance of the plates from each other, and equal to that in a tube whose *radius* is equal to the interval between the plates*.

If a capillary tube be withdrawn from the liquid by which it is wetted, the upper surface will be concave, and at the lower will be a spherical drop, and the height of the suspended column is nearly double the elevation which existed before the tube was withdrawn. The upper surface being concave, the pressure at any point of it is $A-M$.

The lower surface being convex the pressure $A+M'$.

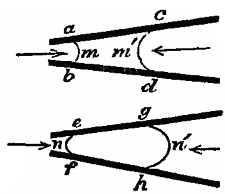
The difference of these, or the force sustaining the column, is $M+M'$; which is in equilibrium with the weight of the column.

* See *Theory of Fluids*, Art. 93.

If the surface of the orifice be not wetted we have, since the curvatures are the same, $M = M'$; the force causing the ascent is therefore $2M$, or the height of the column double that which it was when the tube is immersed.

If the liquid does not wet the tube the pressures at the two curved surfaces balance each other, or their difference is in the direction of gravity; the liquid cannot consequently be sustained.

A small quantity of liquid placed in a cone with its axis horizontal, approaches or recedes from the summit according as the surfaces are concave or convex, that is, as the tube is or is not wetted by the liquid.



The pressures at any points m and m' will be as the forces $A - \dfrac{B}{r}$ and $A - \dfrac{B}{r'}$, if r and r' be the radii of the surfaces, which will be nearly spherical (Art. 215). But ab is more curved than cd, consequently r is less than r'; the quantity subtracted from A is greater in the former case than in the latter; the pressure therefore at m is less than at m', or the drop moves towards the vertex of the cone.

But the surfaces being convex, as ef and gh,

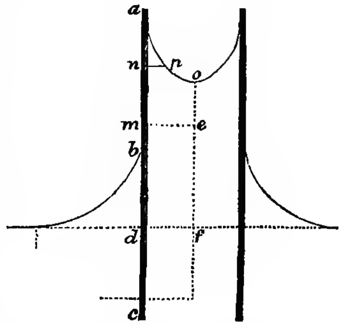
the pressures at n and n' are as $A + \dfrac{B}{r}$ and $A + \dfrac{B}{r'}$,

that is, the greater pressure is at the surface of least curvature, or the drop moves from the vertex.

220. Prop. *Two substances immersed in a liquid within a capillary distance, and both or neither wetted by the liquid, will move together into close contact, and will recede from each other when one is wetted and the other is not wetted.*

Let two substances both of which are wetted by the

liquid be immersed therein, and at such a distance from each other that the liquid is elevated betwixt them as in a capillary tube. These will move towards each other, and come into contact. In examining the forces to which the sides are subject, we may assume that the lateral pressures are equal, and destroy each other at all



corresponding external and internal points of the surface which are in contact with the liquid. We need consequently only examine the forces acting on the portion ab of the mass. Take any point m above the level of the external liquid, and draw any canal meo whereof me is horizontal and eo vertical. Then since the liquid at m is terminated by a plane surface the attraction inwards or the force at m in the direction me is A (Art. 215).

But a particle at m is pressed outwards by the pressure transmitted along the column em, which is that due to the action at the curved surface, and to the gravity of the column oe;

$$\therefore \text{ the pressure outwards at } m = A - M + g \times oe.$$

But $\quad = g \times of$;

$$\therefore \text{ the pressure outwards at } m = A - g \times ef,$$

or, is less than the pressure inwards.

Again, let us consider the forces at any point n. Here the pressure inwards is as before A. Draw n horizontal. Then similarly as before for the action transmitted along pn, we have

$$\text{the outward pressure at } n = A - M.$$

But M will equal $g \times nd$. This then will represent the excess of the pressure from without inwards.

It appears then that for all points of the solid which are in contact only on one side with the liquid, there is a pressure from without inwards proportional to the height of the point above the level of the external liquid; the bodies consequently will move towards each other as if by mutual attraction.

When the solids are not wetted by the liquid, it may be shewn in the same manner that the parts which are in contact only on one side with the liquid, are acted on by forces from without inwards which are not counterbalanced, and the bodies consequently move towards each other as if attracted.

When two solids are immersed, whereof one is wetted and the other is not wetted by the liquid, and the space betwixt them is such that the curvatures of the surfaces just unite, the following takes place.

There is a point of inflexion just at the point of this union, and on the same level as the external surface; at this point the surface is plane. The elevation and depression of the liquid betwixt the bodies is less than the elevation and depression which exists on the outer side; for the depressed liquid tends to diminish the curvature of the elevated liquid, and conversely; so that the curvature of the surface of the liquid against the inner side of each solid, is less than that against the outer side. Whence it follows, that the elevations and depressions being inversely as the radii of curvatures, are less (Art. 214).

From this it will follow, from reasoning similar to the preceding, that each solid is attracted towards the greater curvature; the bodies consequently recede from each other.

221. *Theories of Laplace and Poisson.* In the theory which has been presented in the preceding articles of capillary

phenomena the liquid has been supposed incompressible. This hypothesis, which is perfectly consistent with all observed phenomena, together with the following hypotheses, that there is an attraction of the particles of the liquid for each other, and also a mutual attraction betwixt the particles of the liquid and the particles of the solid, and that these forces are sensible only at insensible distances from the attractive centres, led Laplace to an explanation of the many curious phenomena of capillary action.

Reasoning on these principles Laplace concluded that the attraction of capillary tubes influences the elevation or depression of the liquid only, by determining the inclination of the liquid surface to the contiguous surface of the solid, on which inclination the concavity and convexity of the fluid surface depend, as well as the magnitude of its radius. The concavity or convexity of the surface thus comes to be spoken of as the *cause* of the elevation or depression of the column; for we have seen that the nature of the surface, whether plane, concave, or convex, (Arts. 212, 213), influences the amount of pressure which is exerted at each point thereof.

Poisson, however, does not admit the introduction of the first of these hypotheses, namely, the incompressibility of the liquid; but arguing on the hypothesis of the molecular constitution of bodies, their porosity, and known compressibility, contends that the pressure at any point of the surface arising from the action of the molecular forces, cannot increase with the distance from the plane surface of the liquid without causing a corresponding variation in the distance of the particles near the surface, unless there be also a variation in the quantity of heat producing the repulsive force, for the particles so acted on. There being no such variation in the quantity of heat, the density of the liquid must increase with the distance from the free surface till we reach the limit of the sphere of the sensible action of the particles. Thus Poisson

contends for a superficial variation in the density of the liquid at the surfaces in question: and that corresponding to the modifications in the resultants of the attractive forces to which the bounding surfaces are subject, there will be differences of density imperceptible in themselves, but causing variations in the pressure, from which very sensible effects will result.

Taking then into consideration the compressibility of the liquid, Poisson arrives at the following important conclusion, that "capillary phenomena are due to the molecular action resulting from the calorific repulsion and an attractive force, and modified not only by the form of the surface, as in Laplace's theory, but moreover by a particular state of the compression of the liquid at its superficies." Also, that "the molecular attraction in liquids as well as solids extends further than the calorific repulsion*."

The variation in density near the surface must be extremely rapid; and as experiments have never detected its existence, we have a sort of negative evidence that the superficial stratum in which there is any sensible variation must be extremely minute. If that depth may be neglected in comparison of the sensible activity of the attractive force, Laplace's principles suffice for a theory of capillary action, without being inconsistent with those of Poisson†.

The results obtained by Laplace may be considered as first approximations; and as there do not appear to be any objections to the reasoning in any part of Laplace's Theory, we must for the present leave the more delicate and refined theory of Poisson to the researches of the speculative philosopher. The pursuit of this subject, with an especial view to detecting this variation in density at the surface, is of the

* M. Poisson, *Nouvelle Théorie de l'Action capillaire*, Arts. 30, 31.

† Professor Challis, *Report of British Association for* 1834.

greatest importance, as furnishing evidence of the highest order of the truth of the molecular constitution of bodies.

Professor Challis, in concluding his elaborate reports on the Mathematical Theory of Fluids, writes as follows respecting the views of these two celebrated philosophers: "It does not appear that any exception can be taken to the reasoning in any part of Laplace's Theory; the principles may indeed be objected to on the ground which Poisson takes up, viz. that if the molecular constitution of bodies be admitted, there must be a superficial variation of density, which that theory takes no account of; as however experiment has not yet detected any such variation, and we have no means of assigning the amount of its influence, it would be premature to reject the theory on that ground, especially as the probability is that the effects which this consideration has on the numerical results of the calculations will at all events be small*."

* *Reports of British Association for* 1836.

CHAPTER XVII.

ON SOUND.

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222. The phenomena of sound present illustrations of the laws and properties of elastic fluids, different from any which have been treated of in the preceding pages. Sound, strictly speaking, is an effect or sensation produced on the mind by an action upon the organs of the ear; but in common language, a body is said to sound, when it is in the state to produce the impression or sensation of sound, if a medium suitable for transmitting sound exist between the body and the ear. It is well known that sounds are conveyed to our ears from a distance through the air, but it was not until after the discovery of the air-pump that it was ascertained that the air is the active agent, or vehicle of sound; so that if air be absent, or present only in a very rarefied state, as in the exhausted receiver of an air-pump, sound will not be transmitted. Hauksbee* having suspended a bell in the receiver of an air-pump, found the sound die away by degrees, as the air was exhausted, and revive again as the air was readmitted; and if air be forced in by a condenser, the sound will increase in intensity with the degree of condensation. The diminished intensity of sound in the rarefied atmosphere of an elevated mountain-range is well known.

223. Prop. *Sound is produced by a motion of vibration in the sounding body or source of sound, and is transmitted by a like motion.*

* See *Philosophical Transactions*, A.D. 1705.

One of the most familiar instances of the production of sound is when a bell, as of glass or other material, is struck; and a consideration of what then takes place will point out the nature of the motions with which the production of sound is connected. The particles struck recede from the blow, and having moved to their extreme position in one direction, return in the opposite direction and pass beyond their original position, and continue to vibrate or oscillate about their original position, until they return to a state of rest. On the vibration or oscillation of the particles ceasing, the sound ceases also, and whenever we experience the sensation of sound, the vibrations or oscillations of the particles of the body are going on. Hence it is that bodies or substances such as wool or cotton do not produce sound; their constitution is such that the vibrations essential to the production of sound cannot exist. Thus also extremely hard bodies, as masses of stone, which do not readily admit of a change of form, or the particles of which cannot vibrate in the manner of more elastic bodies, are not usually regarded as sonorous.

Sound so produced by motions of vibration is transmitted by a like motion in the air, namely, by a series of alternate condensations and rarefactions consequent upon the particles of the air partaking of the vibratory motion of the sounding body. These alternate condensations and rarefactions of the air being propagated in all directions, travel onwards, and thus the sound is said to be propagated as a wave; that is to say, the transmission of the sound takes place onwards or continuously in one direction from the source of sound, while the motions of the particles to which that transmission is due take place only through a very small distance. The motion of the wave, as in the case of the wave in water, is very different from the motions of the matter, that is, of the particles of the air or water in which

the wave is propagated*. Sound may be produced by the rapid rotation of a cut card, or the stroke of the lash of a whip; and although the particles of the body from which the sound originates are not in a state of vibration or oscillation, similar to the bell already referred to, the air is in the same state of alternate condensation and rarefaction, that being an essential condition of the transmission of any sudden impulse through an elastic fluid. The air is not the only medium which may become the vehicle of sound; liquids and all elastic bodies, whether fluid or solid, also transmit sound in the same manner, the law of the transmission being modified by the nature of the internal constitution of the body.

224. Prop. *Sound is transmitted in air with a uniform velocity of about* 1100 *feet per second.*

That the transmission of sound does not take place instantaneously, but requires sensible time, is proved by ordinary observation. The report of a gun is heard long after the flash is seen, and the interval is longer the more distant the gun. Sound being transmitted by the propagation of the series of successive condensations and rarefactions, the velocity of sound will be the same as the velocity of transmission of such condensations and rarefactions. This will obviously depend on the character of the medium through which the sound is transmitted, and, in the case of the atmosphere, on the various circumstances affecting its

* The phenomena of waves in water are more readily apprehended than in air, owing to the former being visible to our senses. In the circular waves produced by a stone thrown into still water, and in the waves which roll on to the sea-shore, the particles of the water do not move in the direction in which the *form* travels. The surface of the water is successively elevated and depressed, and the wave travels onward; when the elevation and depression of the particles are interfered with by the shore, the wave breaks. See Webster's *Elements of Physics.*

elasticity and pressure. Considerable discrepancy existed for some time between the velocity of sound, as determined from observation and theory, but this discrepancy was removed when the development of heat during the condensations, and the absorption of heat during the rarefactions (Art. 174), and the corresponding effects on the elasticity of the air, were properly taken into the account; and the velocity may be taken at about 1100 feet per second*.

It may also be mentioned, as the result of observation and theory, that all sounds travel with equal velocity in the same medium; were not this the case, the notes of different musical instruments would not reach the ear in the precise order in which they are played, nor would the component notes of a harmony, in which several sounds of different pitch concur, arrive at once.

225. PROP. *Sound is reflected, the angles of incidence and reflexion being equal.*

The fact of the reflexion of sound is well established by the phenomenon of echoes; but the law according to which it is reflected as established by observation and derived from the principles which are adopted as the basis of the mathematical theory of sound, is that of the equality of the angles of incidence and reflexion. Sound is emitted in all directions from the sounding body or source of sound; the successive condensations and rarefactions constituting a series of spherical waves propagated in succession through space, and moving in the direction of radii from the source of sound: thus sound, in the same sense as heat and light, may be said to be propagated by rays in every direction. The propagation of sound in waves moving as it were from a centre in the direction of radii, follows as a consequence

* See *Theory of Fluids,* Art. 142, and *Encyc. Metropolitana,* Art. *Sound,* 67—70.

from the hypothesis that each particle or molecule of air when disturbed communicates its motion of vibration to the surrounding particles or molecules, and thus becomes a centre of fresh disturbance; whence it comes to pass that sound, after passing through an aperture, is propagated in all directions*.

When the wave by which sound is propagated meets with any obstacle, as a hard or elastic body, it cannot advance; the motion is not however extinguished, but the vibration of the aerial particles is reflected. The obstacle thus becomes the origin of a new vibratory motion in the air, and a centre of fresh waves which cross other waves in the same manner as waves in still water.

The reflecting obstacle thus becoming as it were a fresh source of sound, the position of an observer with reference to this obstacle and the original source of sound is most material, and the result of observation and experience shews that the phenomena of the reflexion of sound may be well expressed by the law that the angles of incidence and reflexion are equal. If an observer be situate midway between two plane surfaces, a sound originated at the same spot will be reverberated from both surfaces and reach him at the same instant and reinforce each other; if he be nearer to one surface than the other, the sound will be confused or double. If the reflecting or echoing surface be concave, the reflected sounds will after reflexion converge towards certain foci or centres, in the same manner as was explained in reference to heat

* Experience shews a remarkable distinction to exist between the spread of sound and light. Sound on passing through a hole is propagated laterally as well as straight forward, whereas light is propagated only straight forward, making a spot on a body placed in front of the hole of the same size as the hole. This results from the fact that the length of the waves of light is less than the diameter of the aperture, whereas the length of the waves of sound will generally be greater than the diameter of the aperture. See Airy's *Undulatory Theory of Optics*, Art. 25.

(Art. 170); so that the observer situate in the centre of a spherical surface, or at the focus of an elliptical surface, will receive a much louder echo than when situate between two plane surfaces.

226. *Illustrations.* The theory of sound as explained in the preceding articles admits of many familiar illustrations, among which the following are especially deserving of notice.

It has been stated that all sounds travel with equal velocity, but the distance to which sound can be propagated so as to be audible depends on the state of the atmosphere and the presence of any good reflecting surface. Thus during a dense fog, and falling rain and snow especially, the free propagation of sound is materially affected. A similar effect is produced by a fresh fall of snow on the ground; but when the surface of the snow has become glazed with a coating of ice, the propagation of sound takes place very readily, as over still water*, or along a smooth wall†.

Effect of pipes. The effect of smooth pipes in preventing the spread and loss of sound is well known. The use of speaking tubes between different apartments affords an illustration of this; and according to an experiment of M. Biot, the lowest whisper was heard from one end to the other of a system of water-pipes of more than 3000 feet in length. A well at Carisbrook Castle, in the Isle of Wight, 210 feet in depth and 12 feet in diameter, is lined throughout with very smooth masonry; a pin dropped from the surface will be

* See *Encyc. Metrop.* Art. *Sound,* 21.

† The whispering-gallery at St Paul's, London, and the echoes at many churches and other buildings, present well-known illustrations; and it may be observed, that whenever a room presents a curved surface, as a dome or vaulted roof, an echo (if the room be of sufficient dimensions) or a confusion of sounds may be expected. The echo at the Menai Bridge in North Wales is particularly deserving of attention. See *Encyc. Metrop.* Art. *Sound,* 35.

heard to strike the water. The disturbance of the air is in these cases confined to a limited mass or column, instead of being communicated and allowed to spread in all directions, and may thus be transmitted onwards to great distances without any considerable diminution of intensity.

Sounding-Boards. The parabolic sounding-board presents one of the most obvious confirmations of the above principles. It is a property of the parabola that all lines from the focus to the surface make the same angle with the perpendicular or tangent to the surface at any point as a line parallel to the axis of the parabola. Hence, if the angles of incidence and reflexion of sound be equal, a ray of sound proceeding from the focus will be reflected in a direction parallel to the axis of the parabola; so that if the mouth of a speaker be in the focus of a parabolic sounding-board, that is, of a surface generated by the revolution of a parabola about its axis, the sound which diverges in spherical waves above his head will all be reflected in the same direction and parallel to the axis of the parabola. Those who are within the ordinary range of the speaker's voice will hear by direct rays of sound, and those at a distance by reflected rays; some persons may hear from both sources, but no confusion will be introduced, since the sound from the two sources, namely, direct and reflected, will appear to come at the same instant. The same instrument will also render a very low whisper at a distance audible, and an ear in the focus of such a sounding-board may hear at the distance of many feet a sound which would scarcely be audible directly at a much smaller distance.

Thunder. The rolling or peals of thunder, and its sudden bursts, are explicable on the same principles. It appears from observation, that under a clear sky the explosion of a cannon is heard as a sharp and single sound; but if the sky be overcast, there will not unfrequently be a long-continued roll, and

occasionally a double sound from a single shot. The continuous peal of thunder may therefore be a series of echoes from different parts of the dense clouds which usually accompany a thunder-storm.

But there is also another cause; the transmission of electricity like light is for all distances within the limits of our atmosphere (Art. 129) instantaneous. The sound, then, consequent on a discharge of electric fluid, will be generated at the same instant through many miles of cloud, but it will travel to our ears with a velocity of about 1100 feet per second (Art. 224); so that it will arrive successively from different points, thus producing a continuous peal or roll of thunder. When the sound arrives from any number of points at the same instant, a sudden burst or clap is the consequence, but in general the sound is continuous, augmented at intervals by echoes so as to cause all those variations in the intensity of the sound which add so much to the grandeur of the scene.

227. *Musical sounds.* The sounds to which the term musical is applied differ from ordinary sounds, or noises, in that the impulses producing the sound are similar in duration and intensity, and recur at equal intervals of time; and the differences in the rapidity with which the impulses are repeated, or the duration of the vibrations, constitutes that comparison of sounds which is technically expressed by the term *tones*. If a musical string, as a metallic wire stretched between two points, be struck, and then left to itself, it will vibrate for a certain period, giving rise to a sound which will be heard at a greater or a less distance; but the character of the sound or the tone will be the same so long as the isochronism of the vibrations is preserved: the extent of the vibration, or excursion of the string and the intensity of the sound, will gradually diminish, but the time of each excursion, and consequently the tone, will be unaltered. If the string be stretched with greater force the vibrations will be more

rapid; the sound will not be the same, but will have a higher tone; the *pitch* will be different*.

228. *Interference of sound.* The phenomena of musical sounds, particularly those known as beats in music, furnish a direct proof of the interference of sound, that is to say, of cases in which one set of aerial vibrations may be added to another so as to increase or diminish the intensity of the resulting sound, according to the state of motion in the molecules of air by which the sound is to be propagated.

229. It would be foreign to the present treatise to enter more into detail respecting the laws of the sounds produced by the vibration of strings or of a column of air, and the phenomena of musical sounds and instruments generally. The vibrations of bodies, whether fluid or solid, are intimately connected with the laws of their internal constitution and the mutual connexion of their component particles; a subject of the greatest difficulty.

The subject of the theory of sound is of great importance when viewed in connexion with the theories of light and heat and electricity, which latter are connected together in a remarkable manner in other respects, as well as in the laws of vibrations and undulations, to which, in common with sound, they must be referred.

One remarkable similarity between the laws of sound and light must be mentioned, namely, the interference of the waves. It is certain that waves of sound may interfere with each other so as to produce comparative stillness, in the same manner as waves of light may interfere with each other so as to produce comparative darkness.

The effect upon the human ear of the impulses producing the sensation of sound is very remarkable: if the number of

* See Webster's *Elements of Physics*.

impulses exceed or fall short of a certain number per second, that is, so to speak, of the limits for each particular ear, all sense or perception of sound is lost. Some persons never heard the cry of a bat, and are altogether insensible to sounds which are painfully shrill to others. The impulses may lose in intensity more than they gain in velocity and consequently not have intensity sufficient to affect the internal organs of the ear, or those organs may be unable to transmit more than a limited number of aerial vibrations or impulses in a given time; but, whatever may be the way in which the sensation of hearing is connected with the vibrations of matter, whether gaseous or liquid, it is not unphilosophical to conjecture that animals exist whose powers of hearing commence where ours terminate, and who may hear sounds of the existence of which we are ignorant; that one medium, as air or water, may convey distinct sounds to different orders of creation, so that there may be whole classes of living and moving beings having no sound in common.

CHAPTER XVIII.

ON WAVES AND TIDES, AND MOTION OF WATER AND TIDAL PHENOMENA IN RIVERS AND ESTUARIES.

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230. The particles of fluids are acted on by gravity, and subject to the same laws of gravity as the particles of solids (Art. 10); a given mass of fluid is subject to the same laws of equilibrium and of motion as a solid; but inasmuch as the particles of a fluid move *inter se*, the fluid mass is subject to a continual change of form, which occasions great difficulties in any attempt to treat the subject mathematically.

The motion of water in a pipe, trough, or channel of uniform section is referable to the laws of gravity and the equal transmission of fluid pressure, and may be the subject of mathematical calculation. The velocity in each case, other things being the same, is due to the head, that is, to the height of the water at one point above its height at another point of the mass or the surface when the water is unconfined, as in an open trough, canal, or river.

Owing to the instantaneous transmission of pressure equally in all directions (Arts. 21—26), an increase or diminution of pressure at any part of the mass of a fluid at rest and free to move, as the addition or abstraction of a small quantity of water in one portion of a pipe containing water and open at each end, will be followed instantaneously by a change in the condition of pressure and by motion throughout the entire mass, as by running over at one end contemporaneously (to the senses) with the introduction of the water at the other

end. Water contained in a pipe constitutes a machine, the parts of which, that is the particles of water, are so intimately connected that the consequent motion and the change in pressure on the introduction or abstraction of a portion of water are to the senses at least contemporaneous, and the motion consequent thereon may be said to be instantaneous. Such transmission, although instantaneous to the senses, is not absolutely so, by reason of the water being slightly compressible; instead therefore of saying that the transmission of pressure is immediate or instantaneous, it may be more correct to say that there is an immediate or instantaneous alteration of pressure throughout the entire mass.

The same law of the instantaneous transmission of pressure, or of change in the condition of pressure, exists in water not confined on all sides; as for instance, if a trough or canal be full of water, the addition or abstraction of a portion of water at any part, as at one end, will cause the running over instantaneously of an infinitesimal small quantity at the other end, the expression instantaneously being qualified by the extent to which water is compressible. When however the water is not inclosed on all sides the transmission of motion is accompanied by phenomena of waves, to the laws of the generation and propagation of which attention must be particularly directed.

231. *A wave is the transmission of an undulation or transference of a form independent of the motion of translation of the mass of the water.*

A clear apprehension of the meaning of the term 'wave of water' is essential to an understanding of the phenomena of tides, especially as exhibited in tidal rivers and estuaries.

In the theory of Acoustics (Art. 223) and Optics the term undulation is frequently employed to express the same as the term wave; the term wave and undulation being used indif-

ferently to express the same thing, namely, a transmission and transference of form or state of particles; such transmission or transference of form being altogether different from the motion of the particles of the fluid in which the transmission or transference takes place. Each wave is but an advancing form, the particles of the water partaking of a vertical motion, as will at once be evident from the motion of a cork, piece of wood, or other floating substance, or a vessel riding at anchor; the particles of water partaking of these vertical motions in succession constitute or occasion the transference of the state of particles exhibited by the travelling wave.

The particles of the water may be conceived to partake of small motions alternately upwards and downwards, and alternately forwards and backwards in reference to the direction in which the wave is travelling, the wave moving continually in the same direction. This oscillatory or reciprocating motion of the particles is sufficient to explain the continuous motion of the shape, form, or arrangement of particles constantly in one direction*.

Thus the continued motion of the wave in a given direction is not the continued motion of the water in that direction, but the continued motion of a shape or form, or of an arrangement of the particles of the water.

232. *Characteristics of waves.* The characteristics of a wave are as follows. Each particle is disturbed in a horizontal as well as in a vertical direction; the state of each particle in advance is, at some future time, the same as the state of each particle in arrear; the motion of each particle is reciprocating or oscillating.

If the same form be transferred at the same rate, the above characteristics are preserved; but if in the course of the motion of the wave an alteration takes place in the form transmitted,

* See *Encyclopædia Metropolitana*, Art. *Tides and Waves* (136—9).

as by an alteration of the linear interval between the crest of two successive waves, or in the shape of the crest, there will have been a corresponding alteration in the motion of the particles.

Thus waves may present another or fourth characteristic; the wave or form in the course of its progress may at one time or place be long and flat, and at another time and place be short and flat, also the rate of transmission or velocity of the wave may alter.

The conditions of the equilibrium and motion of fluids involved in such alterations have reference to the continuity and equal pressure of fluids; if the depth of the water be variable the theory shews that the two conditions of continuity and equal pressure cannot be satisfied in a series of waves the particles of which partake of oscillating motions.

The tidal wave up a river or estuary presents many such changes; a wave 18 feet in height at the entrance of the Bristol Channel will be 30 feet at Swansea, 50 feet at Chepstow, 18 feet at Newnham and 7 feet at Gloucester. In these and similar instances the form of the wave changes in its progress, the change of form and progress of the wave being influenced by the depth, width, and section of the estuary or channel along which it travels.

233. *A wave may be generated by any disturbance of the relative position* inter se *of the particles.*

The peculiar mode of producing the disturbance or undulation in the water, or of genrating the wave, influences or determines to some extent the character, or as it is technically called, the phase of the wave.

A wave may be generated by the sudden introduction or withdrawal of a solid; by the sudden addition or abstraction of a portion of the water itself; by the continuous action of pressure, as of the wind, or in any way by which the relative position of the particles *inter se* is disturbed. The

wave generated by the paddles of a steam-boat presents in passing away the phenomena of a considerable depression, or, as it has been termed, a negative wave*, and is included in the general mathematical expression for a wave by a change of the sign of the coefficient, the one (or negative wave) being a progressive hollow or depression, the other (or positive wave) being a progressive elevation.

One of the simplest modes of generating a wave for the purpose of observation on the laws of its propagation or transmission is by the addition or abstraction of a portion of water to or from water contained in a trough, or by the admission of a portion of water at the bottom, thereby producing an elevation or swell at the surface, which, by reason of the mutual connexion of the particles, will be transmitted throughout the trough.

234. *The propagation of a wave depends on the height, length, and form of the wave, depth of water, and section of containing channel.*

A wave generated in water, mercury, or other liquid contained in a rectangular trough will be propagated or travel along the trough with a velocity varying as the square root of the depth of the water in the trough. Thus, supposing the quantity of water in a trough to be doubled, or the width of the trough containing the water to be reduced one half, the depth being in one case *four* and in the other case *eight*, the velocity in the latter would be to the velocity in the former case as *two* to the *square root of eight* ($2 : \sqrt{8}$); or suppose the depth were four and sixteen respectively, the velocity of transmission or propagation would be in the proportion of two to four.

Theory indicates and experiment confirms the above law of the velocity of transmission of a wave in a channel of uniform

* See experiments by John Scott Russell. Report of 7th Meeting (Liverpool, 1837) of British Association, pp. 417—496.

depth, breadth, and section; but when the channel is not of uniform depth and breadth, and the section variable, the velocity of transmission does not follow so simple a law. For such cases neither theory nor experiment has as yet done much towards ascertaining the law of propagation.

In the experiments above referred to in a trough of rectangular or uniform section, one wave only is travelling over the surface at the same time; but in nature waves follow each other in quick succession. The following general conclusions indicated by theory will be useful in making observations on waves.

In water of a given depth the velocity of different kinds of waves is not the same, but depends on the length of the wave and upon the interval of time at which successive waves follow each other. If the distance between the crests of the waves or the interval of time between successive waves be given, the velocity of the wave will vary with the depth of the water.

When the length of the wave is not greater than the depth of the water, the velocity of the wave depends (sensibly) only on its length, and is proportional to the square root of its length.

When the length of a wave is not less than a thousand times the depth of the water, the velocity of the wave depends (sensibly) only on the depth, and is proportional to the square root of the depth. It is, in fact, the same as the velocity which a free body would acquire by falling from rest, under the action of gravity, through a height equal to half the depth of the water. From these conclusions it will appear that in making observations on waves, or in reasoning thereon, it will be necessary to direct attention to the depth of the water, the height and length of the wave, and the mode of its generation.

The greater velocity of large as compared with the velocity of small waves, also indicated by theory, is consistent with experience.

235. *Reflection, crossing, and interference of waves.* The experiments above referred to on the motion of a wave in a rectangular channel, afford a good illustration of the reflection of waves. The wave having travelled from one end of the channel to the other, will be reflected by the plane end and return back again over the same space without alteration of form. Thus a channel twenty feet in length may be used for determining the velocity of the wave, in lieu of a channel many times that length; inasmuch as a wave in such a channel, observed after reflection 60 times, will really have described 1200 feet; and the observations on its velocity and motion being made at different points of its length, as say 3 points in the length of 20 feet, are equivalent to observations at 180 points in the 1200 feet of its motion. Waves, however produced, observed in a rectangular channel after reflection in the manner described, shew no perceptible difference of velocity.

As in the case of the undulations or waves of air by which sound is transmitted, two waves of water may cross each other without interference, or may interfere in such manner as to obliterate each other and produce comparative stillness; this may be observed in the series of circular waves, diverging from a centre, caused by throwing two stones of equal size into water at the same moment, under favourable circumstances; for such an experiment a series of points may be distinguished, in which the water remains smooth, whilst it is agitated at intermediate parts.

236. *Waves due to action of wind.* The waves produced by the action of the wind upon the surface of water, are those with which we are most ordinarily acquainted. The action of the wind on the surface of the water produces in the first instance very shallow undulations, or increases the undulations already existing. The height of the wave becomes increased

in every part, during which the heads will ordinarily be broken, from the circumstance of the particles at the crest or top of the wave being driven forward in the direction of the wind with too great force for continuous undulation.

These are the phenomena when the sea is rising, but after a time the height of the waves (beginning from the windward shore) will be so much increased as that the power of the wind will merely maintain them in that state without any increase, but for all the sea in advance the wind will still be raising the waves. As the wave necessarily attains that height which corresponds with the height which the wind can just maintain, those waves will no longer be increased though the waves in advance will still be increased. Thus a wind of given intensity, however long it blows, can only raise the waves at a given point to a certain height, which height however will depend upon the distance of that point from the windward shore. The action of the wind will maintain a series of waves, whose elevation beginning at the windward shore is nothing, and goes on increasing successively from wave to wave without remarkable alteration from time to time; when a certain magnitude has once been attained, and when the waves have attained this certain magnitude, their heads will hardly be broken by the action of the wind.

It should be remarked, that in the open sea the waves of large amplitude only are so much increased as to attract attention; the reason of this would appear to be, that when the action of the wind has in some degree increased all the waves, the long protect the short waves from the continuance of its action upon them. And thus the long waves are conspicuous at open sea, not because the short waves are changed into long waves, but because the long waves only are increased conspicuously from the windward shore to the open sea.

Considerable difference of opinion exists as to the height of waves in the ocean under the action of the wind. The

Astronomer Royal inclines to the opinion, that in no case the height of an unbroken wave exceeds 30 or 40 feet*.

237. *Tides in the ocean.* The attraction of gravitation upon the liquid envelope of the earth, in combination with the rotation of the earth about its axis, gives rise to a class of phenomena different in kind from any which have been as yet considered. These phenomena of the ocean tides, generally designated as *the tides*, are as follow:

The surface of the sea at any place is liable to periodical variations in height, which accurate observation has shewn to depend on the relative position of the moon, combined in some degree with that of the sun.

The tides present three orders or classes of phenomena which are separately distinguishable; the first kind occurs twice a day, the second twice a month, and the third twice a year. Every day, about the time of the moon's passing over the meridian, or a certain number of hours later, the sea becomes elevated above its mean height, and at this time it is said to be high water. The elevation subsides by degrees, and in about six hours it is low water, the sea having attained its greatest depression; after this it rises again when the moon passes the meridian below the horizon, so that the ebb and flood occur twice a day, but become daily later and later by about $50\frac{1}{2}$ minutes, which is the excess of a lunar day above a solar one, since $28\frac{1}{2}$ lunar days are nearly equal to $29\frac{1}{2}$ solar ones.

The second phenomenon is, that the tides are sensibly increased at the time of the new and full moon; this increase

* See *Encyc. Metrop.* Art. on *Tides and Waves* (417). The following will be read with interest.—"In H.M.S. *Thetis*, during an unusually heavy gale of wind in the Atlantic, not far from the Bay of Biscay, while between two waves, her storm try-sails were totally becalmed, the crest of each wave being above the level of the centre of her main yard when she was upright between two seas. Her main yard was 60 feet from the water-line." *Voyages of the "Adventure" and "Beagle,"* Appendix, 297, by Captain Fitzroy, R.N.

and diminution constituting the spring and neap tides: the augmentation becomes also still more observable when the moon is in its perigee, or nearest the earth. The lowest as well as the highest water is at the time of the spring tides; the neap tides neither rise so high nor fall so low.

The third phenomenon of the tides is the augmentation which occurs at the time of the equinoxes; so that the greatest tides are when a new or full moon happens near the equinox, while the moon is in its perigee. The effects of these tides are often still more increased by the equinoctial winds, which are sometimes so powerful as to produce a greater tide before or after the equinox, than that which happens in the usual course, at the time of the equinox itself.

The above are the phenomena exhibited in connexion with the position and action of the moon, but the sun also produces a series of effects the same in kind, but different in degree, on account of its greater distance, to those produced by the moon; the tide due to the solar, being about two-fifths of the tide due to the lunar, action.

These tides take place independent of each other, nearly in the same degree as if they were single, and the combination resulting from them is alternately increased and diminished accordingly, as they agree or disagree with respect to the time of high water at a given place. Hence are derived the spring and neap tides; the effects of the sun and moon being united at the times of conjunction and opposition, or of the new and full moon, and opposed at the quadratures, or at the first and last quarters.

The phenomena above described are common to the tides of the ocean, and to the tides as seen on the shores of any continent; the latter however may be designated as derivative tides, that is to say, they are derived from the elevation and depression of the ocean at and about the region of the equator; and although the general phenomena are the same in all cases,

local causes give rise to various peculiarities deserving the attention of the student*.

238. *Tides in the atmosphere.* The attractions of the sun and moon on the fluid envelope of the earth constituting our atmosphere, must occasion tides in the atmosphere having the same general characteristics as tides of the ocean. The greater or less accumulation of the air over any particular portion of the earth, according to the position of the tidal wave and the consequent condensation or rarefaction of the air, produces an increased or diminished weight of the atmospheric column, which should be indicated by corresponding periodical variations or oscillations of the barometer. These effects due to the attraction of the sun and moon are however so inconsiderable in comparison with the disturbances produced in the equilibrium of the atmosphere by other causes, that very careful observations are necessary for their detection; but such observations unequivocally indicate the effect of the lunar action in producing a tide in the atmosphere†.

The effects due to the attraction of the sun on the solar tide in the atmosphere is much more difficult to be detected, not only because it is much smaller, but because the periodical oscillations of the barometer due to solar heat are much greater.

The effect of the variation in the pressure of the atmosphere upon the ocean tide, in producing a greater or less elevation or depression, has been examined with great care and made the subject of calculation ‡.

* The subject of the tides has engaged the attention of Newton, Bernoulli, Laplace, Lubbock, Airy, Whewell, and indeed of all the most eminent mathematicians of modern times.

See treatise on *Tides and Waves* in the *Encyclopædia Metropolitana*, by G. B. Airy, the present Astronomer Royal. See also Lord Brougham's *Analytical View of Newton's Principia.*

† See paper by Colonel Sabine, *Phil. Tr.* 1847, p. 45.

‡ See researches of Sir John Lubbock, *Phil. Tr.* 1837, p. 97; Daussey, *Connaissance des Temps*, 1834; and of Mr Birt, 11th *Report of British Association for the Advancement of Science.*

239. *Tides in rivers and estuaries.* Although the tides in rivers and estuaries present the general characteristics of the phenomena above described, local circumstances produce changes requiring to be particularly noticed.

The elevation or depression of the water due to tidal action at or near the mouth or entrance of a river, or the tidal wave, is transmitted according to the laws already referred to. This transmission or travel of the tide wave is independent of the current of the river. Tides in a river present the following phenomena. The duration of the fall is greater than the rise, that is, the interval from high water to low water is greater than the interval from low water to high water, or, in other words, the ebb is longer than the flood*. It will also be found that the current in a river runs upwards for some time after high water, and downwards for some time after low water, when it again changes its direction and runs upwards.

Any person observing a tidal river, as the Thames from the centre pier of London Bridge, would perceive a current continuing to run upwards after the surface of the water has dropped to the extent of nearly two feet, and to run downward, after the surface of the water has risen to a similar extent.

This fact has given rise to the popular error, on the part of persons viewing this and other rivers from the bank only, of supposing "that it is not high or low water in the centre of the channel until long after it is high or low water at the sides," regardless of the physical impossibility that the water should not preserve a level surface, but stand at one time considerably higher and at another considerably lower than at the shore.

The large quantity of fresh or land water, finding its outlet to sea by the Thames and other rivers, materially affects

* Some apparent exceptions occur in this. For instance, at Portsmouth, owing to the combination of the tides due to the passage by Spithead and the Needles, the tide in Portsmouth Harbour flows 7 and ebbs 5 hours, a fact not without its influence in the maintenance of that important harbour.

the state of high and low water as regards its elevation and depression; but it does not affect the transmission of the tide wave, or state of high and low water, except that the velocity of the current is to be added to or deducted from the velocity of such transmission, according as the direction of the two are the same or opposite: except to this extent,—the velocity of the transmission of the wave is independent of the manner in which the disturbance is made or the wave generated.

240. *The velocity of the tidal wave in a river or estuary is measured by the interval between the time of high and low water at successive places.*

The velocity of propagation of the wave, though nearly independent of the mode of its generation or the nature of the disturbance, is materially affected by local circumstances.

The velocity of the transmission of a wave in a channel rectangular throughout, or contracting and expanding gradually and uniformly, in water 10 feet in depth, is about 18 feet per second; and in water 36 feet in depth would be about 34 feet in a second; a result arrived at by theory in so simple a case, and confirmed by observation. But theory is unable to deal with this subject in the ordinary case of tidal rivers and estuaries, and recourse must be had to observation in particular cases.

In the Thames. If high water occurs on a certain day at Margate, at 12 o'clock, it will occur at Sheerness at 24 minutes past 1; at Gravesend at 15 minutes past 2; and at London Bridge a few minutes before 3; the distance from Margate to London Bridge being about 70 miles. Thus the tidal wave has been propagated, or the state of high water has travelled 70 miles in about 3 hours, or at the rate of more than 23 miles an hour for this lower portion of the river.

Again, the high water which occurs at the London Docks at 4.15, will occur at Battersea Bridge at about 5 o'clock, at

Putney Bridge at $5\frac{1}{4}$, at Kew Bridge at 5.30, and at Teddington Lock at $6\frac{1}{4}$ o'clock. Thus in the upper reaches of the river, obstructed by numerous bridges and other impediments, the tidal wave travels about 20 miles in two hours, or at the rate of about 10 miles an hour*.

Great care must be taken not to confound the velocity of the stream or current on the flood and ebb tide with the velocity of the tidal wave. The stream passes between London Bridge and Putney Bridge on the flood tide in about $6\frac{1}{4}$ hours, and on the ebb tide in about $5\frac{3}{4}$ hours, a distance of $7\frac{1}{2}$ miles; or the velocity of the current either on the flood or ebb is not more than $1\frac{1}{2}$ miles per hour †.

The Mersey, Dee, Severn, Tyne, Clyde, and all other tidal rivers and estuaries, present similar phenomena, materially modified however by local causes. The general characteristics of such phenomena are the same, being due to the same general cause, namely, the elevation and depression of the ocean tidal wave; but each river and estuary presents some features of detail peculiar to itself‡.

In a river of great length there may at the same instant be several tides or states of high and low water at different points along the river; such is recorded as observed in the river Amazons.

241. *Alterations in the range of tide and time of high water.*

Observation confirms that which theory would lead us to expect, namely, that when a river contracts or shoals rapidly,

* See Beardmore's *Hydraulic and Tide Tables* (Weale, London), pp. lxviii—lxxii, lxxi—iii, for much instructive information on this subject.

† See observations of Messrs Rennie and Giles, as recorded in the *Fourth Report of the British Association*, and in Beardmore's *Hydraulic and Tide Tables* (Weale, London).

‡ See Beardmore's *Hydraulic and Tide Tables* for these particulars. Also Webster's *Birkenhead Docks*, for observations on Tides of the Mersey.

the range of tide is increased. In a uniform channel, in consequence of the effect of friction, the range will decrease; there is then some rate of contraction with which the range of tide will be stationary; if the river contracts more rapidly, the range will increase from the preponderating effect of contraction: if it contracts less rapidly, and, *à fortiori*, if it expands, the range will diminish from the preponderating effect of friction. This increase of range or heaping up of the water at the upper end of a tidal river or estuary, is due to the change of form and the momentum of tidal column; the tidal wave meeting with a rapid contraction the water moves on, the momentum being the same in the one as in the other column. Also the range of the tide in passing up a river may increase and decrease successively.

From this also it will be evident, that the low water in the upper part of a tidal river or estuary may be higher than the high water nearer the sea.

The interruption of a channel generally uniform by a barrier, as an elevation of the bottom or the piers of a bridge, is attended with similar alterations in the range of the tide.

Such contraction of channel and increased range of tide may produce a reflex action and recoil, materially affecting the quantity of water entering from the sea; hence in treating a channel artificially, care must be taken not to contract the channel too much. One main object of hydraulic engineering is to increase the quantity and efficiency of the water entering a tidal river or estuary from the sea and the scour on the ebb. By an improvement in the channel the high-water may be raised and the low-water surface depressed, thus increasing the influence of the tidal column; and this raising of the high-water and depression of the low-water surface will be concurrent results. An advance in the time of high water will also result from such improvements; the time of high water

will occur earlier, so that the periods of the flow and ebb may be more nearly equalized, the latter being in general considerably in excess of the former.

242. *Illustrations.*

Various instances may be cited in illustration of the alterations in the range of tide and time of high water.

In the Thames, the mean range at Sheerness is 13 feet, at Deptford 17, at London Bridge about 15, and at Teddington Lock about 2 feet.

In the Severn, a tidal elevation or wave of about 18 feet in height at the entrance of the British Channel, will have a height of 30 feet at Swansea, of 50 feet at Chepstow, of 18 feet at Newnham, and of 7 feet at Gloucester. A wave having a height of 27 feet off Lundy Island, will attain a height of 60 feet at Chepstow; and a wave in the Bay of Fundy, of a height of 8 feet at the entrance of the Bay, will have a height of 25 feet at the Island of Grand Manan, and a height of 75 feet at Chignecto Bay, the alteration in height between these points of the Bristol Channel and the Bay of Fundy being 33 and 17, 50, 67, feet respectively.

A wave having a mean range of about 13 feet near the mouth of the Seine has 20 feet at Havre, 13 feet at Quillebauf, and thence decreases rapidly; and a wave of 15 feet at Cherbourg has a range of nearly 40 feet at St Malo.

In the St Lawrence the range increases from 4 or 5 feet at the mouth to 14 feet at Quebec, after which it dies away.

In the Wash, between Lincolnshire and Norfolk, the range of tide is about 26 feet, but at Yarmouth only 6.

In these and similar cases the elevation of the ocean tidal wave may be regarded as uniform; but the increased elevations or ranges of tide are due to the operation of the momentum, and the change of form impressed by local causes.

243. *The Bore.*

When the tide rises rapidly with a tidal wave of a considerable height (as at Newnham on the Severn, where it rises 18 feet in an hour and a half), a phenomenon called the Bore may be produced under certain circumstances. The Bore is a broken wave advancing with great rapidity and noise. A rapid rise of tide and considerable extent of flat shore near the level of low water, are necessary for its production. The rapid rise of the tide elevating the surface of the water considerably above the flats, the water immediately rushes over them with great velocity, and with a broken front and a great noise. The rise of the water continues after the Bore has passed with apparently unabated rapidity, until the river is full.

This phenomenon is exhibited also in the Seine, the Amazons, the Bays of Chignecto and of Mines, and at the head of the Bay of Fundy.

244. *Equilibrium between descending and ascending columns in tidal rivers.*

The column of descending water is opposed by the column of ascending water, and these balance or come into equilibrium with each other.

The straightening and deepening of the outfall of a river (so successfully effected in the Nene and in the Ouse), while it admits more tidal water as an ascending column, also increases the power of the descending column. The depression or lowering of the low-water surface, and removal of obstructions to the discharge of the descending waters, is equivalent in effect to increasing the natural fall of the river from the highest point at which the tide is felt. Any increase in the fall of the river gives the descending column of fresh water additional velocity and momentum, or greater mechanical power, whereby the equilibrium betwixt the two columns will be established at a point more seaward. This imaginary boundary or point

where the two forces are in equilibrium, will move in the direction of the increased force; for example, when the force of this land-water is increased by a flood, the boundary will travel seawards; and when the tide is aided by a wind from the sea, the boundary will travel upwards*.

245. *Action of waves and currents to transport solid matter.*

The deposit of solid matter below the high-water line on or contiguous to the bed of tidal rivers and estuaries, the variation, travel, and removal of such deposits, are among the most interesting questions of hydraulic engineering.

The matter of such deposit may exist in the water in a state of solution, as it is commonly called, that is, of mechanical suspension, by reason of the specific gravity of the matter and water differing insensibly from each other; or matter of greater specific gravity than water may be transported by, and, as it were, suspended in water, by reason of the agitation of the water and the velocity of its motion.

A very small amount of sensible agitation in the water will retain the matter firstly above referred to in its state of mechanical suspension; and such matter will not be deposited so long as such agitation is continued, which will generally be the case in tidal rivers and estuaries, except in extremely sheltered places. The soft mud of rivers and estuaries is a deposit made under the above circumstances, and which readily resumes its state of mechanical suspension, on being subjected to agitation, similar to that on the cessation of which it was deposited.

The current takes up and transports solid matter of greater or less specific gravity, according to its velocity; depositing

* See *Report* by Robert Stephenson *on the Improvement of the Nene*, in reference to the then (A.D. 1848) much agitated question whether the straightened outfall would bring the salt water more inland. Experience shews the boundary to be taken more to sea.

a portion of such matter at or near any spot where its velocity is checked, thus forming a submerged bank, as in the neighbourhood of piers of bridges, or of any solid obstruction in the water-way, or near the base of a wall, the friction along the face of which reduces the velocity of the film of water contiguous to such wall.

The changing character of the velocity of a current may be observed in the eddies, the slack water, and the reverse or opposite currents which occasionally exist; in all such cases there is a tendency to form a submerged bank or shoal, varying in height and dimensions according to the velocity of the current at the surface of the accumulation. When the forces tending to deposit and to removal at such surface are in equilibrium with each other, no change takes place; the submerged bank or shoal will remain, or decrease, according as the one or other action is in excess.

The wave also sustains matter in suspension, according to the velocity of the oscillating and reciprocating motion of its constituent particles; but the travel of the wave in the proper sense of the term is not accompanied by any translation or transference of such particles. A portion of the water subject to waves, transports, during the filling or emptying of any receptacle, the matter which the agitation keeps in suspension, and such matter on the cessation of the agitation becomes deposited. The particles of solid and of fluid matter form equally part of the wave, but the translation of solid matter is due to the motion of the water in a mass, and not to the travel of the wave.

Just as the subsidence of the wave leads to the deposit of solid matter in suspension, renewed agitation brings the matter so deposited into a state of suspension, and such matter becomes removed from the place of deposit, just in the same manner as it was brought in, by the withdrawal of the water in a state of agitation, and the breaking of the waves. When however

the wave becomes broken, there is a transfer or translation of the particles of solid, as well as of fluid matter.

246. *Harbours, Submerged Beaches.* The application of the above principles is illustrated by the silting of receptacles for water subject to tidal action, as bays of the sea, the entrances of tidal rivers and estuaries, basins and harbours situate on such rivers and estuaries, in each of which a deposit takes place under the action of the causes just referred to. Some peculiarities however deserve to be noticed, both in natural and artificial harbours. The indraught of water on the filling, and the discharge of water on the emptying, is accompanied by the deposit and removal of sand or solid matter, in proportion to the quantity of water entering and discharged; this quantity in some rivers, as the Tyne or the Tees, and other estuaries under similar physical conditions, is very large, as compared with the quantity entering or discharged from the Wear or Bay of Hartlepool, geographically situate betwixt the two former. The variation in the state of such rivers and estuaries is consequently much greater in the former than in the latter case. Similar variations to those taking place daily in the shore of the sea betwixt high and low water, are also taking place in that part of the shore permanently covered with water, that is in the portion of the shore below low water; theory and observation concur in the fact of the periodical motion and fluctuation of such submerged beaches.

In the filling and emptying four times a day of a basin or harbour, situate on a river or estuary subject to tidal influence, the above causes are in operation, exhibiting however results of a less extensive character.

The current passing across the mouth or entrance of a basin or harbour, on the flood or rising tide, at a velocity say of from 4 to 6 miles an hour, will carry along with it in suspension the matter due to its velocity. The water entering the basin or

harbour may not have a velocity of more than $\frac{1}{2}$ a mile an hour; the solid matter capable of being moved, or carried in by such a velocity will be exceedingly small; and no solid matter will enter except what is due to the velocity of indraught; the heavier matter will pass on with the current, and the indraught necessary for filling the basin or harbour will carry with it no other particles of solid matter than those which its velocity keeps in suspension.

Inasmuch as the section of the entrance to such basin or harbour will generally be less than any other section of the basin or harbour, the velocity of the indraught or outdraught will be greater in that than in any other section; the matter will be deposited as such velocity decreases, so that the heavier matter will lie nearest the entrance, and the lighter matter only will be carried further in. During the emptying of the basin on the ebb or falling tide, the velocity of the outdraught and power of removal of the deposited matter will be greatest in the minimum section.

This tendency to remove on the emptying, matter brought in and deposited on the filling of the basin or harbour, will not generally be sufficient to keep the entrance clear, but artificial means, as by dredging or a current, will be required.

In some cases, as for instance Portsmouth Harbour, where from its peculiar situation in reference to the two passages by the Isle of Wight, the tide flows *seven* and ebbs *five* hours, the greater velocity of the efflux than of the influx assists in the maintenance of the Harbour.

247. *Canal wave navigation.* In the navigation of certain canals of small depth, on which boats are drawn by horses, it was accidentally discovered, that if the boat travelled at a certain speed, the resistance to, or tractive force necessary for maintaining the speed of the boat, was less than if the boat travelled at a greater or a less velocity. This excep-

tional case to the general law of the increase of the resistance according to the square of the velocity (152 *b*) is referable to the fact, that a boat, travelling along a canal, at whatever speed, is accompanied by a wave, travelling at the same speed as the boat, whose motion at that particular speed is maintained by the horizontal pressure of the boat upon the water. This, which may be called the *forced* wave, or wave due to the immediate action of the impressed forces, is distinct from the *free* wave, which will travel along the canal at a particular velocity varying as the square root of the depth (Art. 234). If the boat moves more slowly than this free wave, the forced wave precedes the middle of the boat, and the force necessary to keep up the speed of the boat is (in proportion to its velocity) considerable; if the boat moves more rapidly than the free wave, the forced wave follows the middle of the boat, and the force necessary to maintain the speed of the boat is (in proportion to its veloctiy) less than in the former case; but if the boat moves with a velocity equal to, or rather slightly exceeding, the velocity of the free wave it rides with its middle on the top of the wave, and is drawn along by a force much less (in proportion to its velocity) than at lower speeds, or indeed by a force absolutely less than at such speeds*.

* See the facts stated, and results analysed, by John Scott Russell, *Edinburgh Transactions*, Vol. XIV; also *Encyclopædia Metropolitana*, Art. *Tides and Waves* (404—9).

APPENDIX.—TABLES, &c.

I.

SPECIFIC GRAVITIES.

Thermometer 60^{0} *F.*
Barometer 30 inches.
A cubic inch of water weighs 253.17 grains.
A cubic inch of air weighs ·3100117 grains.

Acid, Acetic	1.062
Arsenic	3.391
Arsenious	3.728
Benzoic	0.667
Boracic, crystallized	1.479
Do. fused	1.803
Citric	1.034
Formic	1.116
Fluoric	1.060
Molybdic	3.460
Muriatic	1.200
Nitric	1.271
Do. highly concentrated	1.583
Phosphoric, liquid	1.558
Do. solid	2.800
Sulphuric	1.850
Agate	2.590
Alcohol, Absolute	0.797
Do. highly rectified	0.809
Do. of commerce	0.835
Alum	1.714
Amber from 1.065 to	1.100
Ambergris from 0.780 to	0.926
Amethyst, common	2.750
oriental	3.391
Amianthus from 1.000 to	2.313
Ammonia, aqueous	0.875
Arragonite	2.900
Azure-stone	2.850
Barytes, Sulphate of, from 4.000 to	4.865
Do. Carbonate of, from 4.100 to	4.600
Basalts from 2.421 to	3.000
Beryl, oriental	3.549
Do. occidental	2.723
Blood, human	1.053
Do. crassamentum of	1.245
Do. serum of	1.030
Borax	1.714
Butter	0.942
Camphor	0.988
Caoutchouc, or India rubber	0.933
Carnelion, speckled	2.613
Chalcedony common, from 2.600 to	2.650
Chalk from 2.252 to	2.657
Chrysolite	3.400
Chrystalline Lens of the Eye	1.100
Cinnabar, from Almaden	6.902
Coals from 1.020 to	1.300
Copal	1.045
Coral, red from 2.630 to	2.857
white from 2.540 to	2.570
Corundum	3.710
Cyder	1.018
Diamond, oriental, colourless	3.521
Do. coloured varieties, from 3.523 to	3.550
Diamond, Brazilian	3.444
Do. coloured varieties, from 3.518 to	3.550
Dolomite from 2.540 to	2.830
Dragon's Blood (a resin)	1.204
Ether, Acetic	0.866
Muriatic	0.729
Nitric	0.908
Sulphuric from 0.632 to	0.775
Emerald from 2.600 to	2.770
Euclase from 2.900 to	3.300
Fat of Beef	0.923
Hogs	0.936
Mutton	0.923
Veal	0.934
Felspar from 2.438 to	2.700
Flint, black	2.582
Gamboge	1.222
Garnet, precious from 4.000 to	2.230
Do. common from 3.576 to	3.700

Gases*,— Atmospheric Air	1.000
Ammoniacal	0.590
Carbonic Acid	1.527
Carbonic Oxide	0.972
Carburetted Hydrogen	0.972
Chlorine	2.500
Chlorocarbonous Acid	3.472
Chloroprussic Acid	2.152
Cyanogen	1.805
Euchlorine	2.440
Fluoboric Acid	2.371
Fluosilicic Acid	3.632
Hydriodic Acid	4.340
Hydrogen	0.069
Muriatic Acid	1.284
Nitric Oxide	1.041
Nitrogen	0.972
Nitrous Acid	2.638
Nitrous Oxide	1.527
Oxygen	1.111
Phosphuretted Hydrogen	0.902
Prussic Acid	0.937
Sub-carburetted Hydrogen	0.555
Sub-phosphuretted ditto	0.972
Sulphuretted ditto	1.180
Sulphurous Acid	2.222
Glass, crown	2.520
green	2.642
flint ......from 2.760 to	3.000
plate	2.942
Granite .... from 2.613 to	2.956
Gum arabic	1.452
cherry-tree	1.481
Gunpowder, loose	0.836
shaken	0.932
solid	1.745
Gypsum, compact from 1.872 to	2.288
crystallized, from 2.311 to	3.000
Heliotrope, or Bloodstone, from 2.629 to	2.700
Honey	1.450
Honeystone, or Mellite, from 1.560 to	1.666
Hornblende, common, from 3.250 to	3.830
basaltic, from 3.160 to	3.333
Hornstone.........from 2.533 to	2.810
Hyacinth .... from 4.000 to	4.780
Jasper .... from 2.358 to	2.816
Jet	1.300

Indigo	1.009
Ironstone from Carron	3.281
Do. Lancashire	3.573
Isinglass	1.111
Ivory	1.825
Lapis Nephriticus	2.894
Lard	0.947
Lead, Glance or Galena from Derbyshire ... from 6.565 to	7.786
Limestone, compact, from 2.386 to	3.000
Magnesia, native, Hydrate of	2.330
Do. Carbonate of, from 2.220 to	2.612
Malachite, compact, from 3.572 to	3.994
Marble, Carrara	2.716
white Italian	2.707
black veined	2.704
Parian	2.560
Mastic, (a resin)	1.074
Melanite, or black Garnet, from 3.691 to	3.800
Metals, Antimony	6.702
Arsenic	5.763
Bismuth	9.880
Brass ... from 7.824 to	8.396
Cadmium	8.600
Chromium	5.900
Cobalt	8.600
Columbium	5.600
Copper	8.900
Gold, cast	19.25
Do. hammered	19.35
Iridium, hammered	23.00
Iron, cast at Carron	7.248
Do. bar-hardened, or not	7.788
Lead	11.35
Maganese	8.000
Mercury, solid, 3^0 below 0 of Fahr.	15.61
Do. at 32^0 of Fahr.	13.69
Do. at 60^0 of Fahr.	13.58
Do. at 212^0 of Fahr	13.37
Molybdenum	8.600
Nickel, cast	8.279
forged	7.666
Osmium and Rhodium, alloy of	19.50
Palladium	11.80
Platinum	21.47
Potassium at 59^0 Fah.	0.865
Rhodium	10.65
Selenium	4.300
Silver	10.47
hammered	10.51

It must be remembered that atmospheric air, the barometer at 30 inches, is the medium of reference for the gases and vapours.

Metals, Sodium at 50° Fahr.	0.972
Steel, soft	7.833
tempered	7.816
hardened	7.840
tempered & hardened	7.818
Tellurium, from 5.700 to	6.115
Tin, Cornish	7.291
Do. hardened	7.299
Tungsten	17.40
Uranium	9.000
Zinc ... from 6.900 to	7.191
Mica from 2.650 to	2.934
Milk	1.032
Mineral pitch, or Asphaltum, from 0.905 to	1.650
Mineral Tallow	0.770
Myrrh, (a resin)	1.360
Naphtha from 0.700 to	0.847
Nitre	1.900
Obsidian from 2.348 to	2.370
Oils, Essential—Amber	0.868
Aniseseed	0.986
Carawayseed	0.904
Cinnamon	1.043
Cloves	1.036
Fennel	0.929
Lavender	0.894
Mint, common	0.898
Turpentine	0.870
Wormwood	0.907
Expressed—Sweet Almonds	0.932
Codfish	0.923
Filberts	0.916
Hempseed	0.926
Linseed	0.940
Olives	0.915
Poppyseed	0.939
Rapeseed	0.913
Walnuts, from 0.923 to	0.947
Whale	0.923
Opal, precious	2.114
Common, from 1.958 to	2.114
Opium	1.336
Orpiment from 3.048 to	3.500
Oyster-shell	2.092
Pearl, oriental, from 2.510 to	2.750
Pearlstone	2.340
Peat from 0.600 to	1.329
Peruvian Bark	0.784
Phosphorus	1.770
Pitchstone from 1.970 to	2.720
Plumbago or Graphite, from 1.987 to	2.400
Porcelain, from China	2.384
Sevres	2.145
Porphyry from 2.452 to	2.972
Porphyry, Seltzer	1.003
Proof-spirit	0.923
Pumice-stone ... from 0.752 to	0.914
Quartz from 2.624 to	3.750
Realgar from 3.225 to	3.338
Rock-crystal ... from 2.581 to	2.888
Ruby, oriental	4.283
Sal Gem	2.143
Sapphire, oriental, from 4.000 to	4.200
Sardonyx from 2.602 to	2.268
Scammony of Smyrna	1.274
Aleppo	1.235
Schorl from 2.922 to	3.452
Serpentine from 2.264 to	2.999
Shale	2.600
Silver Glance ... from 5.300 to	7.208
Slate, (drawing)	2.110
Smalt	2.440
Spar, Fluor from 3.094 to	3.791
Do. calcareous, from 2.620 to	2.837
Spar, double-refracting, from Castleton	2.724
Spermaceti	0.943
Spodumene, or Triphane, from 3.000 to	3.218
Stalactite from 2.323 to	2.546
Steatite from 2.400 to	2.665
Steam of water	0.481
Stilbite from 2.140 to	2.500
Strontian, Sulphate of, from 3.583 to	3.958
Do. Carbonate of, from 3.658 to	3.675
Stone, Bristol ... from 2.510 to	2.640
cutlers'	2.111
grinding	2.142
hard	2.460
paving, ... from 2.415 to	2.708
Portland	2.496
Rotten	1.981
Sugar	1.606
Sulphur, native	2.003
fused	1.990
Talc from 2.080 to	3.000
Tallow	0.941
Topaz from 4.010 to	4.061
Tourmaline from 3.086 to	3.362
Turquoise from 2.500 to	3.000
Ultramarine	2.360
Uranite	2.190
Vesuvian from 3.300 to	3.575
Vinegar from 1.013 to	1.080
Water, distilled	1.000
sea	1.028
of Dead Sea	1.240
Wax, bees'	0.964
White	0.968
Shoemakers'	0.897
Whey, Cows'	1.019

Wine, Bordeaux	0.993
Burgundy	0.991
Constance	1.081
Malaga	1.022
Port	0.997
White Champagne	0.997
Wood, Alder	0.800
Apple-tree	0.793
Ash	0.845
Bay-tree	0.822
Beech	0.852
Box, French	0.912
Dutch	1.328
Brazilian, Red	1.031
Campeachy	0.913
Cedar, Wild	0.596
Palest	0.613
Indian	1.315
American	0.561
Cherry-tree	0.715
Citron	0.726
Cocoa-wood	1.040
Crab-tree	0.765
Cork	0.240
Cypress, Spanish	0.644
Ebony, American	1.331
Indian	1.209
Elder-tree	0.625
Elm-tree	0.671
Filbert-tree	0.600
Fir, Male	0.550
Female	0.498

Wood, Hazel	0.600
Jasmin, Spanish	0.770
Juniper-tree	0.556
Lemon-tree	0.703
Lignum Vitæ	1.333
Linden-tree	0.604
Mastick-tree	0.849
Mahogany	1.063
Maple-tree	0.750
Medlar	0.944
Mulberry, Spanish	0.897
Oak-heart, 60 years old	1.170
Olive-tree	0.927
Orange-tree	0.705
Pear-tree	0.166
Plum-tree	0.785
Pomegranate-tree	1.351
Poplar-tree	0.383
Do. White Spanish	0.529
Quince-tree	0.705
Sassafras	0.482
Vine	1.327
Walnut	0.681
Willow	0.585
Yew, Dutch	0.788
Spanish	0.807
Knot of 16 years old	1.760
Woodstone ......from 2.045 to	2.675
Zeolite ......from 2.073 to	2.718
Zircon ......from 4.385 to	4.700

II.

ELASTIC FORCE, TEMPERATURE AND VOLUME OF STEAM AND WATER IN CONTACT.

Pressure on a Square Inch, including the Pressure of the Atmosphere.		Elastic Force in		Temperature in Degrees of			Volume of Steam compared with the Volume of Water.
		Inches of Mercury.	Metres of Mercury.	Fahrenheit.	Reaum.	Cent.	
lbs.	kilog.						
14.7	6.668	30.00	.762	212.0	80.0	100.0	1700
15	6.80	30.60	.778	212.8	80.4	100.4	1669
16	7.26	32.64	.829	216.3	81.9	102.4	1573
17	7.71	34.68	.880	219.6	83.3	104.2	1488
18	8.16	36.72	.932	222.7	84.7	105.9	1411
19	8.62	38.76	.984	225.6	86.0	107.6	1343
20	9.07	40.80	1.037	228.5	87.3	109.2	1281
21	9.52	42.84	1.089	231.2	88.5	110.7	1225
22	9.98	44.88	1.140	233.8	89.7	112.1	1174
23	10.43	46.92	1.192	236.3	90.8	113.5	1127
24	10.88	48.96	1.244	238.7	91.9	114.8	1084
25	11.34	51.00	1.296	241.0	93.0	116.1	1044
26	11.79	53.04	1.348	243.3	93.9	117.4	1007
27	12.25	55.08	1.400	245.5	94.9	118.6	973
28	12.70	57.12	1.452	247.6	95.8	119.8	941
29	13.15	59.16	1.503	249.6	96.7	120.9	911
30	13.61	61.21	1.555	251.6	97.6	122.0	883
31	14.06	63.24	1.607	253.6	98.5	123.1	857
32	14.51	65.28	1.659	255.5	99.8	124.2	833
33	14.97	67.32	1.711	257.3	100.1	125.2	810
34	15.42	69.36	1.763	259.1	100.9	126.2	788
35	15.87	71.40	1.814	260.9	101.7	127.2	767
36	16.33	73.44	1.866	262.6	102.5	128.1	748
37	16.78	75.48	1.918	264.3	103.2	129.1	729
38	17.23	77.52	1.970	265.9	104.0	129.9	712
39	17.69	79.56	2.022	267.5	104.7	130.8	695
40	18.14	81.60	2.074	269.1	105.4	131.7	679
41	18.59	83.64	2.126	270.6	106.0	132.6	664
42	19.05	85.68	2.178	272.1	106.7	133.4	649
43	19.50	87.72	2.229	273.6	107.4	134.2	635
44	19.96	89.76	2.281	275.0	108.0	135.0	622
45	20.41	91.80	2.333	276.4	108.6	135.8	610
46	20.86	93.84	2.385	277.8	109.2	136.6	598
47	21.32	95.88	2.437	279.2	109.9	137.3	586
48	21.77	97.92	2.489	280.5	110.4	138.1	575
49	22.22	99.96	2.541	381.9	111.1	138.8	564
50	22.68	102.00	2.592	283.2	111.6	139.6	554
51	23.13	104.04	2.644	284.4	112.2	140.2	544
52	23.59	106.08	2.696	285.7	112.8	140.9	534

53	24.04	108.12	2.748	286.9	113.3	141.6	525
54	24.49	110.16	2.800	288.1	113.8	142.3	516
55	24.95	112.20	2.852	289.3	114.4	142.9	508
56	25.40	114.24	2.903	290.5	114.9	143.6	500
57	25.85	116.28	2.955	291.7	115.4	144.3	492
58	26.31	118.32	3.007	292.9	116.0	144.7	484
59	26.76	120.36	3.059	294.2	116.5	146.1	477
60	27.21	122.40	3.111	295.6	117.2	146.4	470
61	27.67	124.44	3.163	296.9	117.7	147.2	463
62	28.12	126.48	3.215	298.1	118.3	147.8	456
63	28.57	128.52	3.266	299.2	118.8	148.4	449
64	29.03	130.56	3.318	300.3	119.2	149.1	443
65	29.48	132.60	3.370	301.3	119.7	149.6	437
66	29.93	134.64	3.422	302.4	120.2	150.2	431
67	30.39	136.68	3.474	303.4	120.6	150.8	425
68	30.84	138.72	3.526	304.4	121.1	151.3	419
69	31.29	140.76	3.577	305.4	121.5	151.9	414
70	31.75	142.80	3.629	306.4	122.0	152.4	408
71	32.20	144.84	3.681	307.4	122.4	153.0	403
72	32.66	146.88	3.733	308.4	122.8	153.6	398
73	33.11	148.92	3.785	309.3	123.2	154.1	393
74	33.56	150.96	3.837	310.3	123.7	154.6	388
75	34.02	153.02	3.889	311.2	124.1	155.1	383
76	34.47	155.06	3.940	312.2	124.5	155.7	379
77	34.93	157.10	3.992	313.1	124.9	156.2	374
78	35.38	159.14	4.044	314.0	125.3	156.7	370
79	35.83	161.18	4.096	314 9	125.7	157.2	366
80	36.29	163.22	4.148	315.8	126.1	157.7	362
81	36.74	165.26	4.199	316.7	126.5	158.2	358
82	37.19	167.30	4.252	317.6	126.9	158.7	354
83	37.65	169.34	4.303	318.4	127.3	159.1	350
84	38.10	171.38	4.355	319.3	127.7	159.6	346
85	38.55	173.42	4.407	320.1	128.0	160.1	342
86	39.01	175.46	4.459	321.0	128.4	160.6	339
87	39.46	177.50	4.511	321.8	128.8	161.0	335
88	39.91	179.54	4.563	322.6	129.2	161.4	332
89	40.37	181.58	4.615	323.5	129.6	161.9	328
90	40.82	183.62	4.666	324.3	129.9	162.4	325
91	41.27	185.66	4.718	325.1	130.3	162.8	322
92	41.73	187.70	4.770	325.9	130.6	163.3	319
93	42.18	189.74	4.822	326.7	131.0	163.7	316
94	42.64	191.78	4.874	327.5	131.3	164.2	313
95	43.09	193.82	4.926	328.2	131.6	164.8	310
96	43.54	195.86	4.977	329.0	132.0	165.0	307
97	44.00	197.90	5.029	329.8	132.4	165.4	304
98	44.45	199.92	5.081	330.5	132.7	165.8	301
99	44.90	201.96	5.133	331.3	133.0	166.3	298
100	45.36	204.01	5.185	332.0	133.3	166.7	295
110	49.89	224.40	5.703	339.2	136.5	170.7	271
120	54.43	244.82	6.222	345.8	139.5	174.3	251
130	58.97	265.23	6.740	352.1	142.3	177.8	233
140	63.50	285.61	7.259	357.9	144.8	181.1	218
150	68.04	306.03	7.778	363.4	147.3	184.1	205
160	72.57	326.42	8.296	368.7	149.6	187.1	193
170	77.11	346.80	8.814	373.6	151.8	189.8	183
180	81.65	367.25	9.333	378.4	153.9	192.4	174
190	86.18	387.60	9.851	382.9	156.0	194.9	166
200	90.72	408.04	10.370	387.3	157.9	197.4	158

III.

ELASTIC FORCE (IN ATMOSPHERES) AND TEMPERATURE OF STEAM AND WATER IN CONTACT.

Elasticity of Steam, the pressure of the atmosphere being 1.	Corresponding temp. in deg. of Fahrenheit.	Elasticity of Steam, the pressure of the atmosphere being 1.	Corresponding temp. in deg. of Fahrenheit.
1	212°	13	380.66°
1½	234	14	386.94
2	350.5	15	392.86
2½	263.8	16	398.48
3	275.2	17	403.83
3½	285	18	408.92
4	293.7	19	413.78
4½	300.3	20	418.46
5	307.5	21	422.96
5½	314.24	22	427.28
6	320.36	23	431.42
6½	326.26	24	435.56
7	331.7		
7½	336.86	25	439.34
8	341.78	30	457.16
9	350.78	35	472.73
10	358.88	40	486.59
11	366.85	45	499.14
12	374	50	510.6

The above Table shews the results of the Committee of the French Academy of Sciences consisting of Arago, Dulong, Geraud and Prony. The Atmosphere is measured by a column of mercury of 29.922 inches (0.76 metre), which is adopted in France as the mean height of the barometer at the surface of the sea.

The last six temperatures in the Table are deduced by calculation. See *Ann. de Chim. et Ph.* XLIII. 74.

IV.

TEMPERATURE AND ELASTIC FORCE OF VAPOUR FROM 32° F. TO 85° F.

Temperature.	Inches of Mercury.	Temperature.	Inches of Mercury.
212 F.	30.	212 F.	30.
32	0.200	59	0.507
33	0.207	60	0.524
34	0.214	61	0.542
35	0.221	62	0.560
36	0.229	63	0.578
37	0.237	64	0.597
38	0.245	65	0.616
39	0.254	66	0.635
40	0.263	67	0.655
41	0.273	68	0.676
42	0.283	69	0.698
43	0.294	70	0.721
44	0.305	71	0.745
45	0.316	72	0.770
46	0.328	73	0.796
47	0.339	74	0.823
48	0.351	75	0.851
49	0.363	76	0.880
50	0.375	77	0.910
51	0.388	78	0.940
52	0.401	79	0.971
53	0.415	80	1.01
54	0.429	81	1.04
55	0.443	82	1.07
56	0.458	83	1.10
57	0.474	84	1.14
58	0.490	85	1.17

See Dalton's paper on Evaporation in the 5th Vol. of the *Memoirs of the Manchester Society*, page 585; also Thomson's *Heat and Electricity*, 186, 3rd edition, for the elastic force of vapour at other temperatures. By means of this table (see Art. 196) the quantity of vapour capable of existing in the atmosphere at a given temperature may be ascertained.

THE END.

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